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► **To cite this version:**

Mira Kaissar Abboud, Ali Khenchaf, Philippe Pouliguen. Gaussian beams formalism for high frequency scattering problems by metallic targets. 2021 IEEE Conference on Antenna Measurements & Applications (CAMA), Nov 2021, Antibes Juan-les-Pins, France. pp.109-112, 10.1109/CAMA49227.2021.9703614 . hal-03648191

HAL Id: hal-03648191

<https://ensta-bretagne.hal.science/hal-03648191v1>

Submitted on 14 Sep 2024

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Gaussian beams formalism for high frequency scattering problems by metallic targets

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Abstract—The aim of this paper is to apply the Gaussian beam formalism for computing high frequency wave propagation problems. Unlike standard ray theory, Gaussian beams method overcome the difficulties of singular regions. The performance of the method is illustrated with several numerical examples.

Index Terms—Gaussian beams, radar cross section, scattering, asymptotic approximations, high frequency

I. INTRODUCTION

The optimization of radar performance has led to the development of signal processing methods that take into account the characteristics of the targets and the parasitic medium. In parallel, a complementary approach to detection has been developed for recognition of radar signatures, allowing the identification of an object among a group of objects according to particular characteristics known a priori. These lines of work have allowed the development of new radar applications such as remote sensing, obstacle avoidance and meteorology. In the military domain, these axes are also exploited to fight against the effects of target signature reduction.

The radar equation links the characteristics of the transmitting system, the observed medium (target) and the receiving device within a single formulation. In this equation, the radar cross section (RCS) is the quantity that contains all the information on the interaction between waves and target. The estimation of the RCS requires the evaluation of the electromagnetic field scattered by any surface and thus the resolution of the propagation equation whatever is the medium. This can be done either by an exact method or by an approximate method. As their name indicates, the exact methods allow to determine exactly the general solution of the propagation equation. However, they require to compute the general solution of second degree equations, differential or integral [1]. Moreover, their numerical versions generally require significant computer resources (memory and calculation time). Approximate methods are generally used in electromagnetism for frequencies above 1 GHz. For these frequencies, the numerical versions of asymptotic methods allow to obtain a solution close to reality in a relatively short time compared to the exact methods. However, this implies the consideration of some simplifying assumptions, limiting their range of validity. These methods are widely used to evaluate the propagation of a field in an inhomogeneous medium as well as to estimate the RCS of non-canonical targets.

Early approaches for prediction of RCS in high-frequency electromagnetic-analysis environment involved the use of the ray method due to the relative simplicity of the formulas and

algorithms that arise. However, the classical ray theory has a fundamental drawback: it is not applicable for calculations in regions where the ray field becomes non regular, i.e. where caustics arise. In realistic problems we virtually always have to deal with singularities, i.e., disruptions of regularity. To circumvent this problem, a number of methods have been proposed in which the wave field is constructed as a superposition of regular waves: the method of edge waves developed by Klem-Musatov [2], the Gaussian beam summation method [3] and its special case called Maslov seismograms [4]. These methods are universal, as the calculation procedure is the same for regular points and for caustic vicinities.

The Gaussian beam method as a method for the computation of wave fields in the high-frequency domain was first proposed in 1981 by Popov [5]. In this paper, we apply the GBM in scattering problem for radar target and compare the method with another asymptotic method, The Physical Optic approximation.

The rest of the paper is organized as follows. In section II, we first review the Gaussian beam model for the computation of high frequency waves. We then derive the general formulation of the GBM at a point situated in the effective vicinity of the receiver. In section III, we present and discuss different choices of initial parameters necessary for the computation. Next, in Section IV, we apply the GBM in scattering problem. Numerical examples of RCS computations are performed. Finally, we summarize our conclusions in section V.

II. GAUSSIAN BEAM METHOD, BASIC FORMULATION

The Gaussian beam approach to solve the problem of wave propagation is to obtain a local paraxial solution to the exact wave equation. We choose a local reference frame linked to the ray with unit tangent vector \vec{t} and unit normal vector \vec{n} (cf. “Fig. 1”). \vec{n} must point everywhere to the same side of the ray. The coordinate s measures the arclength along the ray from an arbitrary reference point, and n represents a length coordinate in the direction perpendicular to the ray at s .

A. Overview on Gaussian beam theory

Consider the Helmholtz equation for a two-dimensional homogeneous medium:

$$\nabla^2 u(x, z, w) + \frac{w^2}{c^2(x, z)} u(x, z, w) = 0 \quad (1)$$

where u is the electric or magnetic field, w is the angular frequency and $c(x, z)$ is the wave velocity at the point (x, z) .

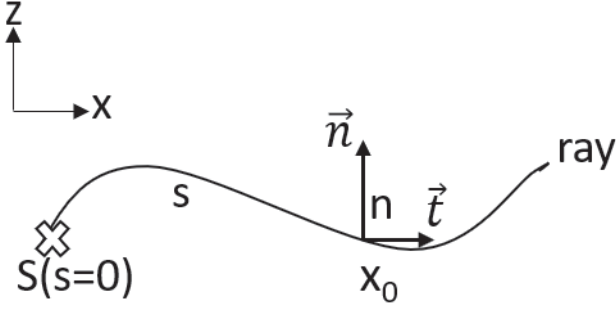


Fig. 1. Definition of the local reference frame.

Gaussian beam wavefield computation uses high frequency asymptotic approximation to transform the Helmholtz equation into a parabolic wave equation in ray-centered coordinates (s, n) [6] [7] [8], as follows:

$$\frac{2i}{c} W_{,s} + W_{,\nu\nu} - \frac{1}{c^3} \nu^2 c_{,nn} W = 0 \quad (2)$$

The relation between u and W is expressed by the equation $u = \sqrt{c(s)} W(s, \nu)$ with $\nu = w^{1/2} n$.

We are looking for a solution of the form:

$$W(s, \nu) = A(s) e^{\frac{i}{2} \nu^2 \Gamma} \quad (3)$$

where $\Gamma = \Gamma(s)$ is complex-valued function. By replacing (3) in (1) and separating real and imaginary parts, we obtain the following two equations:

$$\begin{cases} \Gamma_{,s} + c\Gamma^2 + \frac{1}{c^2} c_{,nn} = 0 \\ A_{,s} + \frac{1}{2} cA\Gamma = 0 \end{cases} \quad (4)$$

We introduce again a new variable q such that:

$$\Gamma(s) = \frac{1}{cq} q_{,s} \quad (5)$$

where $q = q(s)$. We obtain:

$$cq_{,ss} - c_{,s}q_{,s} + \nu_{,nn}q = 0 \quad (6)$$

Equation (6) splits into a system of two first order equations:

$$\begin{cases} \frac{dq}{ds} = cp \\ \frac{dp}{ds} = 0 \end{cases} \quad (7)$$

This system is called a dynamic ray tracing system. p and q can be evaluated as the dynamic ray tracing results. Hill [9] conveniently designated the initial values of p and q to be:

$$\begin{cases} q(s_0) = \frac{ww_0^2}{c} \\ p(s_0) = \frac{i}{c} \end{cases} \quad (8)$$

w_0 is the beam width at the frequency w . By going back to the different variable changes, the wavefield $u(s, n, \omega)$ associated with the raypath is computed by evaluating the function [10] [11]:

$$u(s, n, w) = \left(\frac{c}{q(s)} \right)^{1/2} e^{iw[\tau(s) - \frac{1}{2} \frac{p(s)}{q(s)} n^2]} \quad (9)$$

where $\tau(s) = \int_0^s v^{-1}(s) ds$ the integral evaluated along the ray $r(s)$. Equation (9) is an asymptotic local paraxial solution of the Helmholtz wave equation concentrated close to the ray

path. The second term in the exponential determines that the wavefront is a parabola and that the amplitude profile is a Gaussian function. The higher the frequency is, the sharper the parabolic wavefront and the narrower the Gaussian amplitude profile. Solutions of the form (9) are called Gaussian beams if the following two assumptions are satisfied along the ray:

- $\text{Im}(p/q) > 0$
- $|q| \neq 0$

The first assumption guarantees the concentration of the solution close to the central ray. Second, quantity q describing the geometrical spreading must be nonzero and finite along the ray; thus Gaussian beams, unlike the conventional ray solutions, are nonsingular even at caustics of the wavefield.

B. Calculation of the amplitude at the receiver

Gaussian beams represent high frequency asymptotic solutions of the wave equation. Since the wave equation is linear, the point source wavefields can be expanded into a series of solutions concentrated close to the rays, the Gaussian beams, as:

$$u(P) = \int_{\delta} \Phi(\phi) u_{\phi}(s, n) d\delta \quad (10)$$

where $u(P)$ is the solution at the receiver and $u_{\phi}(s, n)$ is the solution of the beam with take-off angle ϕ , evaluated at P . δ is a cone with a vertex angle ϕ :

$$d\delta = \sin \phi d\phi d\theta \quad (11)$$

with $\theta \in [0, 2\pi]$. The common procedure for estimating the complex-valued weighting function $\Phi(\phi)$ is to compare integral (10) with the exact solution of the wave equation in homogeneous media for frequency $w \rightarrow \infty$ [6]. By geometric considerations, we can then express s and n as a function of ϕ and r , the distance from the source to the receiver. "Fig. 2" shows the geometry of the problem.

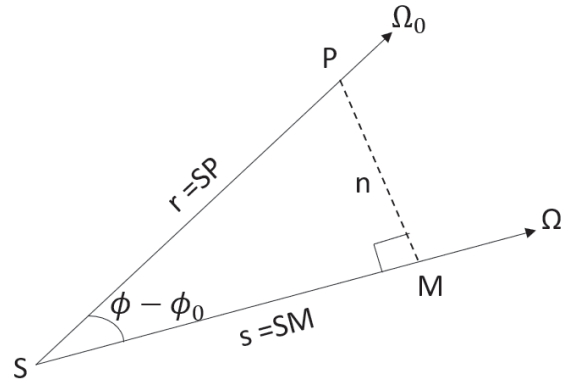


Fig. 2. Sketch to illustrate the geometry of the problem.

Let us consider $s = r \cos(\phi - \phi_0)$ and $n = r \sin(\phi - \phi_0)$. Replacing p , q and τ by their expressions in (9) leads to:

$$u^{GBM}(P) = \int_{\delta} \Phi(\phi) \frac{c^{3/2}}{\sqrt{(ww_0^2 + icr \cos(\phi - \phi_0))}} e^{iw \left[\frac{r \cos(\phi - \phi_0)}{c} + \frac{1}{2} \frac{r^2 \sin^2(\phi - \phi_0)}{ww_0^2 + icr \cos(\phi - \phi_0)} \right]} d\delta \quad (12)$$

Equation (12) is evaluated by a numerical quadrature with regular increment $\Delta\phi$:

$$u^{GBM}(P) = 2\pi \sum_{k=1}^N \Phi(\phi_k) u_{\phi_k} \sin \phi_k \Delta\phi \quad (13)$$

III. ACCURACY OF GAUSSIAN BEAM METHOD

The evaluation of the wavefield by the GBM is influenced by different types of errors due to the approximation and the choice of parameters that specifies the properties of Gaussian beams. Identifying these errors allows us to deal with some of them in order to improve the accuracy of our method. Among these errors, one can distinguish:

- Error in initial data: this error represents how well the exact boundary data is approximated by a superposition of Gaussian beams
- Taylor expansion error: the second error is due to the fact that A and Γ are not computed globally (e.g., (3)), and only their derivatives on the central beams are computed. It is therefore necessary to approximate their values around the central beams by Taylor expansions
- Discretization error: this error results from the fact that the superposition integral is replaced by a discrete summation of beams (e.g., (13))
- Beam waist of the Gaussian beam : a gaussian beam can be fully described by his beam waist w_0 . A good choice of this computation parameter allows to obtain results with the required accuracy.

Here, we will only concentrate on the discretization and the beam waist of Gaussian beam.

To study the efficiency of the GBM, we have applied it to a number of model problems. In this section we present the results of some numerical experiments. At first, we consider a point source and a receiver on the central ray. For homogeneous medium, denoting r the distance to an observation point P , the ray theory approximation of the field is given by the Green's function:

$$u(P) = \frac{e^{iwr/c}}{4\pi r} \quad (14)$$

“Fig. 3” compares the modulus of the wavefield calculated with the ray theory approximation with those computed by GBM for three beam widths w_0 : λ , 2λ , and 10λ , according to the distance on either side of the receiver. This comparison made at 10 GHz and for $N = 61$ Gaussian beams shows that the initial beam waist must be chosen optimally to ensure a good accuracy. Unlike the amplitude of the ray theory which is unbounded at the caustic ($r = 0$), the amplitude of GBM is bounded and increases as w_0 increases.

The modulus of the wavefield computed by the ray theory approximation and by GBM for different beams number are shown in “Fig. 4”. As can be seen, the solution obtained by the GBM converges to the solution obtained by the ray theory approximation as the number of Gaussian beams increases. As in “Fig. 3”, “Fig. 4” shows that GBM is insensitive to singularities.

IV. GBM IN SCATTERING PROBLEM

In this section, we perform two numerical tests and employ the GBM described in Section 2 to compute the RCS of

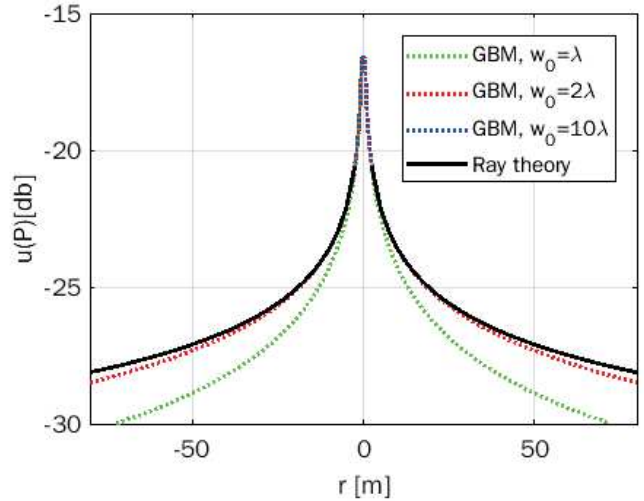


Fig. 3. Absolute value of the wavefield u computed by GBM with different beam waists and by the ray theory.

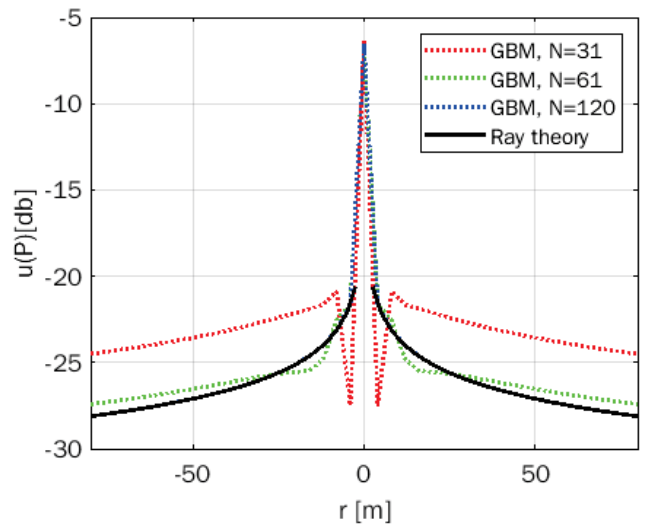


Fig. 4. Absolute value of the wavefield u computed by GBM with different number of beams and by the ray theory.

metallic targets. In both tests, the input data are chosen as follows: $f = 10$ GHz, $w_0 = 10\lambda$ and $N = 200$.

We consider two metallic targets : a circular disk of radius $r = 0.15$ m and a flat plate of sides $2.5\lambda \times 2.5\lambda$. Each target is illuminated by a horizontally polarized plane wave. The GBM is tested in monostatic configuration. The transmitter elevation angle varies from $\phi = 0^\circ$ to $\phi = 85^\circ$. We use the standard Physical Optics (PO) solution computed using FEKO, a computational electromagnetic solver, as a reference solution.

“Fig. 5” shows the RCS of the rectangular plate. Note that the GBM result match the PO solution very well in the specular region. One can see that GBS technique is superposed to the PO solution.

“Fig. 6” shows the RCS of a circular flat plate of radius $r = 0.15$ m. Due to the circular symmetry, the RCS is only aspect angle dependent [12]. For normal incidence, the RCS

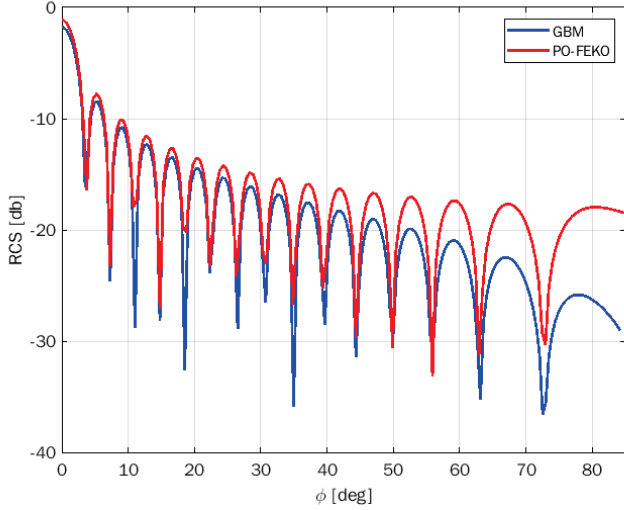


Fig. 5. RCS of a flat plate computed using GBM and PO approximation.

for a circular disk, σ is:

$$\sigma = \frac{4\pi^3 r^4}{\lambda^2} \quad (15)$$

where λ is the wavelength. For non normal incidence, the RCS for any linearly polarized incident wave is given by:

$$\sigma = \frac{\lambda r}{8\pi \sin \phi (\tan(\phi))^2} \quad (16)$$

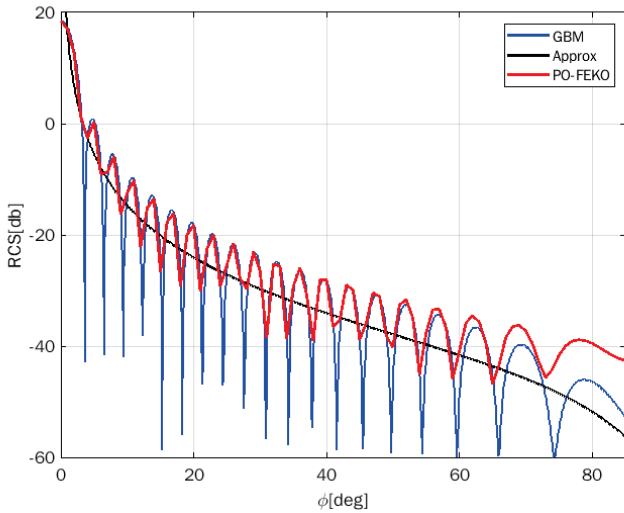


Fig. 6. RCS of a circular disk computed using GBM, PO and the approximation formulas.

As can be seen, the PO and GBM results are in good agreement. However, a worse match with the approximation solution (e.g., (15) and (16)) is observed.

V. CONCLUSION

This paper is devoted for computing RCS of targets using the Gaussian beam method in a monostatic configuration. The total field at the receiver is represented by an integral over Gaussian beams propagating in the neighbour of the receiver.

We investigate the sensitivity of GBM to the initial computation parameters: beams waist and beams density around the observation point. Simulation with canonical targets is shown and will be extend to complex target.

ACKNOWLEDGMENT

The authors wish to thank the DGA-AID (Direction Générale de l'Armement / Agence de l'Innovation de Défense, France) for its support of the ASLESCIM project, where this work is in progress.

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