# Study of the entanglement Rate Influence in a Quantum Illumination Radar Scheme: Information Theory Approach

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***Abstract*—The conventional radar theory relies on the classical electromagnetic wave theory to run. On the contrary, a quantum radar will relie on the quantum mechanic theory to run. A quantum radar that uses the quantum illumination based on photon entanglement enables to get a better detection sensitivity. In this paper, we highlight the influence of the propagation environment on the quantum illumination radar scheme. Our approach consists in describing the quantum information evolution inside the radar and the entanglement rate with a mathematical parameter which represents the interaction strength of the propagation medium on the quantum system used by the radar.**

***Index Terms*—quantum illumination, entanglement rate, quantum radar, decoherence, discord**

## I. Introduction

Since the beginning of 2000's, the rising of the quantum information technologies leads to new great applications in communication sciences like quantum cryptography and superdense coding [1]. The objective of these new technologies is to surpass their classical counterparts in the fields of communication at distance. Quantum information technologies can also be exploited to do remote detection. Then, the idea of quantum radar has made its way and it is still being seriously studied [2].

A quantum radar relies on the quantum mechanic principles to improve the detection sensitivity compared to a conventional radar. Thus, the term "quantum radar" is still a global description that includes several different visions. The idea of a Quantum Illumination Radar has been introduced by Seth Lloyd in 2008 [3]. Such a radar relies on the Quantum Illumination (QI) which consists in using the entanglement between two photons to provide an enhancement of the detection sensitivity in a noisy environment. And despite the entanglement is lost because of the photon-medium interaction, called the decoherence, we observe a detection sensitivity enhancement. Therefore, there might be a quantum information that survived to the decoherence phenomenon to explain the quantum advantage.

In his article [3], S. Lloyd worked in the optical regime where the decoherence is far greater than in the microwave regime, more suitable for a radar application. The microwave regime is already used in the conventional radar theory because of its good transmission in earth atmosphere [4]. Logically, the attention is directed towards microwave photons [5] to further study the Quantum Illumination process in the perspective of a quantum radar.

Hence, our problematic aims to study a Quantum Illumination radar scheme taking into account the propagation medium influence on the quantum information inside our radar scheme. To do that, we take a similar Quantum Illumination Radar scheme as S. Lloyd to study the process in function of a parameter $\Gamma$ depicting the photon-medium interaction strength. Our objective is to describe the quantum system used inside the radar scheme in function of $\Gamma$ and to compute the quantum information evolution as the entanglement rate evolution. Doing that could be interesting to see a link between the quantum information and the detection sensitivity. Thus, we use a system of two qubits entangled in polarization. We express the quantum state of the pair as a function of the parameter $\Gamma$.

For this study, the paper is organized as follow. The sec.II presents the physical concepts involved and the methodology used for the study. The sec.III shows the simulation results with a critical analysis about the results. Then, the sec.IV is devoted to the conclusion.

## II. Physical Principles and Methodology

In this part, we describe the Quantum Illumination Radar scheme used for calculations. Next, we focus on the propagation model and how to compute the information theory quantities to study the radar.

### A. The Radar Equation for the Quantum Illumination Radar

The Quantum Illumination process inside the radar consists in using two entangled photons in polarization to probe a region of space. Fig.1 gives a scheme of this process in three steps. The first step consists in creating the pair of entangled photons. Here, we assume to use a entangled state of two qubits/photons in a pure state $|\Psi^-\rangle_{AS} = (|01\rangle - |10\rangle)/\sqrt{2}$ in the eigenstate basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. In quantum mechanics, a quantum system in a pure state can be written as a state vector i.e. $|\Psi^-\rangle_{AS}$ while it is not possible for a mixed state. An entangled state is a purely quantum phenomenon where a quantum system exhibits strong quantum correlations. We call one photon "A" for ancilla and the other is called "S" for signal. In the second step, the photon A is kept inside a perfect cavity while the photon S is sent

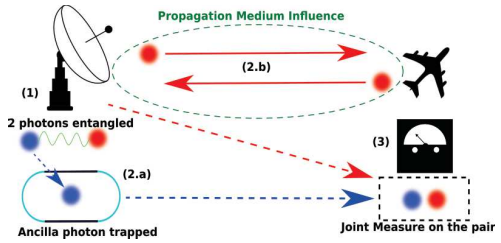


Fig. 1. Illustration of the Quantum Illumination Radar. The QI process is divided in three main parts. In (1), we create the pair of maximally entangled photons. The blue photon refers to the system A and the red photon refers to the system S. The second step is composed of two parts. In (2.a), the blue photon is trapped in a cavity with perfect mirrors. With this approximation, we assume that the blue photon cannot be absorbed inside the cavity. We do not any measure on the blue photon. In (2.b), the red photon is sent towards the target and it interacts with the atomes and molecules of the propagation medium. Next, the red photon is reflected by the object and it comes back to the receiver. In (3), the system AS is gathered to perform a state measurement. This measure represents the basis of a decision strategy which consists to determine the presence or the absence of the object.

towards a region of space where there is probably an object. If the photon S is reflected by an object, it comes back to the receiver of the radar where we make a joint measurement of the quantum state of the photon pair AS. Among these three steps, we highlight the four main parts of the quantum radar chain : emission, reception, propagation, and target.

These four parts permit to get a quantum radar equation-like as for a conventional radar [6]. However in our device, we emit only one photon S to make a detection, not a classical electromagnetic wave. Consequently, if we lose the photon S because of absorption or scattering in the propagation medium, we lose all information. From now, we write the radar equation of the QI radar in (1) as a probability law for the photon S to come back to the receiver after its previous emission.

$$P_{QI} = P_{\text{emission}} \cdot P_{\text{propagation}} \cdot \frac{\sigma_Q}{4\pi R^2} \cdot P_{\text{reception}} \qquad (1)$$

Where $P_{\text{emission}}$ is the emission probability of the photon S. $P_{\text{propagation}}$ is the probability of the photon to propagate before be absorbed or scattered. This probability depends on the medium crossed and it takes the form of a exponential law:$P_{\text{propagation}} = \exp(-C \cdot (x/R)^2)$. $x$ is the distance covered, $R$ is the radar-target distance and $C$ is a constant depending of the propagation medium supposed homogenous and isotrope. The term $\sigma_Q/(4\pi R^2)$ represents the reflection probability for the object to reflect the photon S towards the receiver. The photon-target interaction is described with the Quantum Radar Cross Section (QRCS), noted $\sigma_Q$, which depends strongly on geometrical parameters (see [2] [7] [8]). And $P_{\text{reception}} = \mathcal{A}_{\text{sensor}} P_{\text{sensor}}$ is the reception probability where we assume to use a quantum sensor with an effective cross section $\mathcal{A}_{\text{sensor}}$ with a probability $P_{\text{sensor}}$ to detect the photon close to one.

Thus, the quantum radar equation in (1) is a probability law equivalent to the conventional radar equation that represents an energy balance between emission and reception [6]. In the QI radar, we use two photons but we emit only single-photons. Therefore, the radar equation is the same for a single-photon radar. Finally, the main interest of (1) is to know if the photon can survive along the propagation path. But the detection process does not include only the presence or the absence of the photon S. The keypoint is the particular information conveyed by the two entangled photons. The propagation medium has an impact on the photon S. Then, we focus on the information evolution linked to the propagation path.

### *B. The propagation model*

In this part, we introduce a propagation model related to the QI radar scheme similar to in the Lloyd's article [3]. We refer to a binary decision strategy with two hypothesis $H_0$ and $H_1$ linked to the detection of particular quantum states:entangled or not. The hypothesis $H_0$ corresponds to the absence of a target. As the target is absent, we detect only thermal photons from the environment. The quantum system AS has lost all its entanglement. The hypothesis $H_1$ corresponds to the presence of the target. We receive the photon S after its interaction with the target. The system AS is still entangled. However, the propagation medium has an influence on the quantum state S. And as the quantum state AS exhibits strong quantum correlations, the propagation medium has also an impact on the whole quantum state AS.

We focus our attention on the hypothesis $H_1$ where we receive the photon S after its propagation. We can get two possible quantum states due to the propagation medium. First, we can have a maximally entangled state. This state can be defined with a state vector $|\Psi^-\rangle_{AS}$ and it has the maximum entanglement possible. In second, we can get a mixed state. This state is mixed, it can not be written as a state vector but it can be defined with a density matrix $\hat{\rho}_{AS}^{(sep)}$ containing the quantum state information. By the way, we do the same for the entangled state that gives $\hat{\rho}_{AS}^{(e)} = |\Psi^-\rangle\langle\Psi^-|_{AS}$. Introducing a parameter $\gamma \in [0,1]$ that enables to pass from the entangled state to the mixed one, we write the (2).

$$\hat{\rho}_{AS} \rightarrow \gamma\, \hat{\rho}_{AS}^{(e)} + (1-\gamma)\, \hat{\rho}_{AS}^{(sep)} \qquad (2)$$

Where we define the maximally mixed separable state $\hat{\rho}_{AS}^{(sep)} = \hat{I}_{AS}/4$ with $\hat{I}_{AS}$ the identity matrix of the system AS. The longer the quantum state AS interacts with the propagation medium, the closer to one is $\gamma$. Then, at final, we will get a maximally mixed state. To model this behavior, we define $\gamma = e^{-\Gamma x}$ with the parameter $\Gamma$ representing the interaction strength between the photon S and the propagation medium over the distance $x \in [0, +\infty[$. We put also the condition $\Gamma \in [0,1]$. But the propagation medium does not only destroy the entanglement on AS, it changes also the quantum state AS acting locally on S. The quantum correlations make evolve the quantum state AS by this local action on S while A remains untouched. In quantum mechanics, this propagation medium action is represented by a local measurement performed on the subsystem S without acting on A.

Therefore, the local action of the propagation medium can be represented by a measurement operator $\hat{X}$ depending of the parameter $\Gamma$. This operator does act only on the system S, it takes the form of (3).

$$\hat{X} = \hat{I}_A \otimes \sum_{i=0}^{1} \hat{X}_i \qquad (3)$$

Where $\hat{I}_A$ is the identity matrix on the system A and $\sum_i \hat{X}_i$ are the local measure operator on the system S verifying the unitarity relation with the normalization $K = e^{-2\Gamma x} + e^{2\Gamma x}$. We get (4).

$$\hat{X}_0 = \hat{X}_0^\dagger = \begin{pmatrix} e^{\Gamma x}/K & 0 \\ 0 & e^{-\Gamma x}/K \end{pmatrix} \tag{4a}$$

$$\hat{X}_1 = \hat{X}_1^\dagger = \begin{pmatrix} e^{-\Gamma x}/K & 0 \\ 0 & e^{\Gamma x}/K \end{pmatrix} \tag{4b}$$

The propagation medium acts like a partial local measurement so we apply only one operator $(\hat{I}_A \otimes \hat{X}_0)$ on the system AS which projects progressively the qubit state $\hat{\rho}_S$ on its eigenstate $|0\rangle_S$.

To perform our study, we apply the (4a) to (2) since it describes the evolution of our system AS initially entangled and pure to a system mixed and separable. Equation (2) enables us to define a Werner state (see [9] [10]). Such a quantum state is a mixture of a maximally entangled pure state $\hat{\rho}_{AS}^{(e)}$ and a maximally mixed separable state $\hat{I}_{AS}/4$ like in (2). It gives the density matrix in (5).

$$\begin{pmatrix} \frac{(1-\gamma)e^{2\Gamma x}}{2K} & 0 & 0 & 0 \\ 0 & \frac{(1+\gamma)e^{-2\Gamma x}}{2K} & -\frac{\gamma}{K} & 0 \\ 0 & -\frac{\gamma}{K} & \frac{(1+\gamma)e^{2\Gamma x}}{2K} & 0 \\ 0 & 0 & 0 & \frac{(1-\gamma)e^{-2\Gamma x}}{2K} \end{pmatrix} \tag{5}$$

We note this matrix $\hat{\rho}_{AS}^{(W)}$. This Werner state depends on $\gamma$ by definition and it depends on $\Gamma$ by the application of the operator $\hat{X}_0$. Besides, we apply the measurement operator on the pure state $\hat{\rho}_{AS}^{(e)}$ to make the comparison with the Werner state. We write the density matrix in (6).

$$\hat{\rho}_{AS} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{e^{-2\Gamma x}}{K} & -\frac{1}{K} & 0 \\ 0 & -\frac{1}{K} & \frac{e^{2\Gamma x}}{K} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{6}$$

Note it is physically impossible for a quantum state to remains pure when it interacts with a medium (like atmosphere). Equation (5) represents both actions of the propagation medium while the (6) shows only the partial measurement effect without affect the state configuration. In sec.III, we use both states to understand the quantum information evolution during propagation in the QI radar scheme. To compare (5) and (6) evolutions, we use also the state purity defined by $P(\hat{\rho}_{AS}) = \mathrm{Tr}\{(\hat{\rho}_{AS})^2\}$.

In quantum mechanics, a density matrice $\hat{\rho}_i$ contains all information about a quantum state $i$. Then, we use the density matrice as a calculus basis to estimate the quantum state information. This is the subject of sec.II-C.

### *C. The Information Theory Tools*

In this section, we focus on the quantum information and the classical information of the system AS. To quantify it, we must compute the quantum correlations of the system as its classical correlations. Hence, we have to use the information theory [1]. We are also interested in the entanglement rate calculation. Finally, in the case where several pairs of entangled photons are used, we define the Batthacharyya and the Chernoff bounds to discriminate the both quantum states $\hat{\rho}_{AS}^{(e)}$ and $\hat{\rho}_{AS}^{(sep)}$ of (2).

The Quantum Information Theory (QIT) is the quantum extension of the Classical Information Theory (CIT). In CIT, we use probability laws $p_X$ where $X$ is a random variable to compute the information conveyed by a system. This information is represented by the Shannon entropy $H_X(x) = -\sum_{x\in X} p_x \log p_x$. In QIT, we use the density matrices $\hat{\rho}$ as the analog of the probability laws. And the quantum entropy called the von Neumann entropy is defined as $\mathcal{S}(\hat{\rho}) = -\mathrm{Tr}\{\hat{\rho}\log(\hat{\rho})\}$ where $\mathrm{Tr}\{.\}$ is the trace operation instead of a sum in CIT. From now, we define different information quantities to describe our quantum state AS.

In CIT, for two random variable $X$, $Y$, we define the classical mutual information in (7) in two forms in (7a) and (7b) that are equivalent.

$$I(X,Y) = H(X) + H(Y) - H(X,Y) \tag{7a}$$

$$I(X,Y) = H(X) - H(X|Y) \tag{7b}$$

Where $H(X|Y) = H(X,Y) - H(X)$ is the classical conditional entropy. The quantum extension of the mutual information is naturally the quantum mutual information in (8) for a bipartite state AS.

$$\mathcal{I}(\hat{\rho}_{AS}) = \mathcal{S}(\hat{\rho}_A) + \mathcal{S}(\hat{\rho}_S) - \mathcal{S}(\hat{\rho}_{AS}) \tag{8a}$$

$$\mathcal{I}(\rho_{\hat{A}S}) = \mathcal{S}(\hat{\rho}_A) - \mathcal{S}(\hat{\rho}_A|\hat{\rho}_S) \tag{8b}$$

The calculation of quantum correlations inside a quantum system uses (8a) to quantify the total amount of correlations and (8b) to quantify the classical correlations using the measurement theory [11]. Actually, we use von Neumann measurements $\{\hat{M}_i\}$ with projectors of rank one that verify $\sum_i \hat{M}_i = \hat{I}$. The classical correlations are defined by $\mathcal{J}$ in (9).

$$\begin{aligned} \mathcal{J}(\hat{\rho}_{AS})_{\{\hat{M}_S^{(i)}\}} &= \mathcal{S}(\hat{\rho}_A) - \mathcal{S}(\hat{\rho}_A|\hat{\rho}_S)_{\{\hat{M}_S^{(i)}\}} \\ &= \mathcal{S}(\hat{\rho}_A) - \sum_{i=0}^{1} p_i \mathcal{S}(\hat{\rho}_A^{(i)}) \end{aligned} \tag{9}$$

$\{\hat{M}_S^{(i)}\}$, $i = 0, 1$, are the projectors on the subsystem S. Where $p_i = \mathrm{Tr}\{(\hat{I}_A \otimes \hat{M}_i)\hat{\rho}_{AS}(\hat{I}_A \otimes \hat{M}_i)\}$ and $\hat{\rho}_A^{(i)} = \mathrm{Tr}_S\{(\hat{I}_A \otimes \hat{M}_i)\hat{\rho}_{AS}(\hat{I}_A \otimes \hat{M}_i)\}/p_i$. Using the total amount of correlations of (8a) and the classical correlations of (9), we define the quantum correlations of the system AS in (10) called the quantum discord.

$$d(\hat{\rho}_{AS}) = \mathcal{I}(\hat{\rho}_{AS}) - \sup\left\{\mathcal{J}(\hat{\rho}_{AS})_{\{\hat{M}_S^{(i)}\}}\right\} \tag{10}$$

$\sup\{.\}$ is the supremum over all measurements performed on S. The quantum discord represents global quantum correlations that we must not confuse with the quantum entanglement which is a quantum phenomenon showing strong quantum correlations [11]. To compare the entanglement and the discord, we have also to estimate the entanglement inside the system.

Quantum entanglement is a purely quantum phenomenon that does not have a classical counterpart [12]. Any quantum system that can not be written in a product state of its subsystems is entangled. A product state is a tensor product of several quantum subsystems:$\bigotimes_i \hat{\rho}_i$. For example, $\hat{\rho}_{AS}^{(e)} \neq \hat{\rho}_A \otimes \hat{\rho_S}$ where both subsystems are qubits with $\hat{\rho}_i = \hat{I}/2$. There exists several entanglement measures and their utilization depends on the operational context [11]. Here, we take a

quite common entanglement measure called the entanglement of formation with the concurrence of Wootters [13].

The entanglement of formation consists in calculating the binary entropy $h(x) = -x\log(x) - (1-x)\log(1-x)$ with $x = (1+\sqrt{1-\mathcal{C}^2})/2$ where $\mathcal{C}$ is the concurrence of Wootters. It is defined as $\mathcal{C}(\hat{\rho}_{AS}) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$ where the $\lambda_i$ are the eigenvalues in decreasing order of the matrix $R = \sqrt{\sqrt{\hat{\rho}_{AS}}\tilde{\rho}_{AS}\sqrt{\hat{\rho}_{AS}}}$ with $\tilde{\rho}_{AS} = (\sigma_y \otimes \sigma_y)(\hat{\rho}_{AS})^*(\sigma_y \otimes \sigma_y)$. $(\hat{\rho}_{AS})^*$ is the conjugate of $\hat{\rho}_{AS}$. $\sigma_y$ is the second Pauli matrix among $\{\sigma_x, \sigma_y, \sigma_z\}$.

All the quantities defined are suited to follow the quantum information for one photon pair. But in a QI radar scheme, we should use several pairs to make a detection process. And to know the reliability of the detection using these multiple photon pairs, we can make an estimation of the error probability of detection $P_{\text{error}}$ provided by the Batthacharyya bound and by the Chernoff bound. These two quantities enable to discriminate the two quantum states $\hat{\rho}_{AS}^{(e)}$ and $\hat{\rho}_{AS}^{(sep)}$ when we emit the quantum state AS several times. Both quantities are defined in (11).

$$P_{\text{error}} \leqslant \frac{1}{2}e^{-M\cdot\xi_C} \leqslant \frac{1}{2}e^{-M\cdot\xi_B} \tag{11}$$

Where the coefficients $\xi_B$ for Batthacharyya and $\xi_C$ for Chernoff are defined in (12) and $M$ is the number of copy of the initial quantum state created.

$$\begin{cases} \xi_B &= -\log\left(Tr\{\sqrt{\hat{\rho}_{AS}^{(sep)}}\sqrt{\hat{\rho}_{AS}^{(e)}}\}\right) \\ \xi_C &= -\log\left(\min_{0\leqslant\alpha\leqslant 1}\left(Tr\{(\hat{\rho}_{AS}^{(sep)})^{\alpha}(\hat{\rho}_{AS}^{(e)})^{1-\alpha}\}\right)\right) \end{cases} \tag{12}$$

The Batthacharyya bound is bigger than the Chernoff bound but easier to calculate because we do not have to do a minimization over $\alpha$. Equations (11) and (12) represent the discrimination of both contributions of (2) when they are equiprobable.

Using all these notions, we perform simulations using the density matrices in (5) and (6) to compute the quantum correlations and the entanglement rate.

## III. Simulation Results And Discussion

We performed two simulations for the information evolution from $\hat{\rho}_{AS}^{(W)}$ in (5) and $\hat{\rho}_{AS}^{(e)}$ in (6).

The Fig.2 shows the entanglement rate $\mathcal{E}$ and the quantum discord $d$ as functions of the distance $x$ for several $\Gamma \in [0,1]$. As said in sec.II-B, the state AS remains pure despite the local action on $\hat{\rho}_S$. So calculating the state purity, we obtain : $P = \text{Tr}\{(\hat{\rho}_{AS})^2\} = 1\ \forall x \geqslant 0$. We observe that the entanglement and the discord have exactly the same behavior while their definitions are definitively different. In fact, as we start as a maximally entangled quantum state, we got the maximum of correlations possible so the entanglement is of course maximal but it also the case for the quantum correlations:$d(x=0) = 1$ bit. Although, they are two kinds of quantum correlations, they have the same behavior over $x$. We can explain it by the state purity that is still maximal to one. We see it looking at the density matrix in (6), the state configuration is not affected by the medium action. And naturally, we see also that the closer to one is the parameter $\Gamma$, the faster the entanglement decreases as the quantum discord. If the medium has a strong action on $\hat{\rho}_S$, we lose all quantum correlations faster.

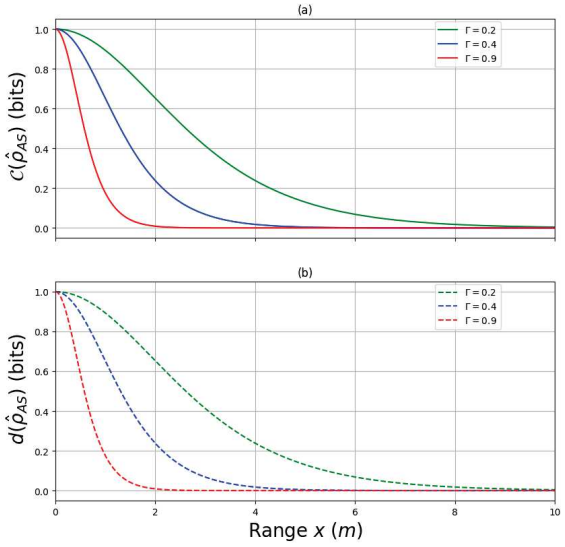


Fig. 2. Graphic of the entanglement rate $\mathcal{E}$ and the quantum discord $d$ for the maximally entangled state $\hat{\rho}_{AS}$. The parameters $\Gamma$ tested are $\Gamma = 0.2, 0.4, 0.8$. The graphic (a) is the entanglement of formation as a function of the distance $x$, calculated with the concurrence of Wooters. The graphic (b) is the quantum discord as a function of the distance $x$ with the projection measurements performed on the system S.

The $\hat{\rho}_{AS}$ state evolution is physically impossible but it still be interesting. An open quantum system is a quantum system that interacts with another system which we do not have any control. Such an open system evolves into a mixed state due to the interaction. As it is not the case in Fig.2, we see the same evolution for quantum correlations and entanglement. Now, we do the same calculations for the state $\hat{\rho}_{AS}^{(W)}$.

In Fig.3a, we plotted with a logarithmic scale the entanglement rate $\mathcal{E}$ and the quantum discord $d$ of the state $\hat{\rho}_{AS}^{(W)}$ as functions of the distance $x$ for three values of $\Gamma \in [0,1]$. In parallel, in Fig.3b, we plotted the purity of $\hat{\rho}_{AS}^{(W)}$ as a function of $x$. Contrary to Fig.2, we observe that the entanglement and the discord have a different behavior. First, the entanglement rate decreases monotonously and it tends to zero during the propagation. The entanglement decay is also faster than for the pure state in Fig.2. Second, the quantum discord has two steps. At the beginning, for small values of $x$, the discord decreases as the entanglement rate until an inflection point. From this inflection point, the discord increases smoothly until stabilize to the value $d = 0.5$ bit while the entanglement rate decreases exponentially. The parameter $\Gamma$ that represents the photon-medium interaction strength controls how fast are the steps of increasing-decreasing. The closer to one is the parameter $\Gamma$, the faster the entanglement decreases as the discord. We note also that the closer to one is $\Gamma$, the lower the stabilization of the discord to $0.5$ bit. We recall that $\hat{\rho}_{AS}^{(W)}$ evolves due to two actions of the propagation medium. The first action is symbolized by the parameter $\gamma = e^{-\Gamma x}$. The longer is the propagation over $x$ for a certain $\Gamma$, the greater the possibility to get a maximally mixed separable state. $\gamma$ translates the evolution of our Werner state from a maximally

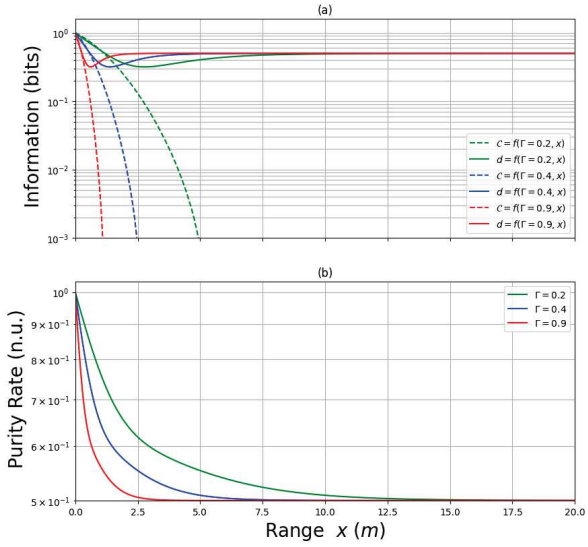


Fig. 3. Graphic of the entanglement rate $\mathcal{E}$ and the quantum discord $d$ for the Werner state $\hat{\rho}_{AS}^{(W)}$. Graphic (a):graphic of the entanglement rate $\mathcal{C}(\hat{\rho}_{AS}^{(W)})$ and the quantum discord $d(\hat{\rho}_{AS}^{(W)})$ as functions of $\Gamma x$. Graphic (b):Purity $P = Tr\{(\hat{\rho}_{AS}^{(W)})^2\}$ of the Werner state over the distance $x$. The values $\Gamma$ tested are $\Gamma = 0.2,\ 0.4,\ 0.9$.

entangled state to a maximally mixed separable one. So there is a moment where we have both contributions that means our system AS is partially entangled and partially mixed. The entanglement rate shows that we are partially entangled and the purity $P < 1$ for $x > 0$ in Fig.3b. The second action consists in the partial local measurement action on the state $\hat{\rho}_S$. It projects the qubit state $\hat{\rho}_S$ on the its eigenstate $|0\rangle_S$. The quantum correlations existing inside the system AS make evolve the state $\hat{\rho}_{AS}^{(W)}$. That produces as for the first propagation medium action the entanglement decay. It is the same situation than in Fig.2. The quantum discord undergoes also these two actions. But contrary to entanglement it does not tend to zero because of the transition of our state $\hat{\rho}_{AS}^{(W)}$ from a maximally entangled pure state $\hat{\rho}_{AS}^{(e)}$ to a maximally mixed separable state $\hat{I}_{AS}/4$. The non zero discord comes from the mixed nature of our final quantum state and we get $d = 0.5$ bit.

From the analysis of Fig.3 compared to Fig.2, we see that under the influence of the propagation medium we keep at least a little part of the quantum correlations. The resilience of this part of quantum information enables us to say a remote detection would be possible. It follows the Lloyd's analysis in [3] where only pure entangled states and separable states were considered. Lloyd considered that even if entanglement was immediatly destroyed by the medium, a reliable detection was possible meaning that some quantum information was kept somewhere. Using the operator of (4a) to progressively change the quantum state AS shows that some quantum correlations survived to the model applied in this paper. Such a case where the quantum state changes progressively under the medium influence is close to using microwave photons in atmosphere. Indeed, the photon-atmosphere coupling is weaker for microwave frequencies than for optical frequencies. Therefore, if the entanglement is instantly destroyed at optical frequencies, it will be not for microwave frequencies.

However, we are aware that the model used is really simple compared to the interactions possible between a photon and the molecules of the atmosphere. We have phenomena of wave scattering and wave absorption that are really important in conventional radar theory [4]. The same phenomena are important for single photons. Futhermore, our model assumed that the photon is not lost during the propagation. Such an assertion is unrealistic as soon as we consider scaterring and absorption. Finally, the parameter $\Gamma$ is only a mathematical parameter and it does not depict a well defined physical quantity.

By the way, a QI radar should use more than one pair of entangled photons to do a detection. That means we should do multiple times the same QI process described in Fig.1. As we use both quantum states $\hat{\rho}_{AS}^{(e)}$ and $\hat{\rho}_{AS}^{(sep)}$ in (2) that correspond to the situation $H_1$ described in sec.II-B, we discriminate these two states where the first one has the maximal amount of quantum correlations while the second one has the minimal quantum correlations. To do it we use the Batthacharyya bound and the Chernoff bound of (11) and (12). Here there is not any dependence on $\Gamma$. Thus, for only one single-photon emission i.e. $M = 1$, we have the two error probabilities $P_e^{(\mathrm{C})} = 0.125 \leq P_e^{(\mathrm{B})} = 0.25$. These probabilities decreases monotonously as we do multiple emissions as showed by Fig.4.

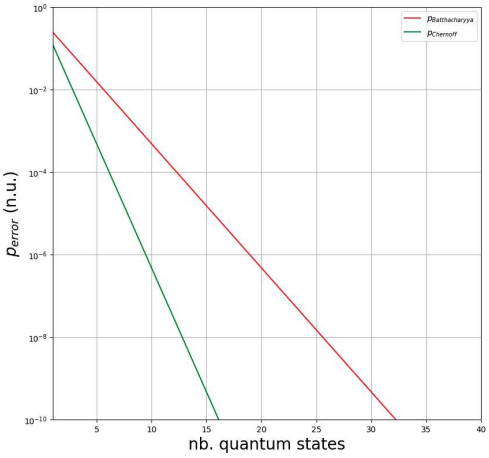


Fig. 4. Graphic of the Batthacharyya bound $p_{\mathrm{Batthacharyya}}$ and of the Chernoff bound $p_{\mathrm{Chernoff}}$ as functions of the number of copies of the initial quantum state.

The error probabilities of detection of the Batthacharyya bound remains bigger than the Chernoff bound as $M$ grows up. Thus, for $M = 10$ number of quantum system copies, we reach $P_e^{(\mathrm{B})} < 10^{-2}$ and $P_e^{(\mathrm{C})} < 10^{-6}$ that represent quite small errors to discriminate reliably both extremal states. Hence, for big $M$, we are able to realiably discriminate the state with the maximal quantum correlations with state with minimal correlations.

Finally, the simulations of quantum information evolution performed with the states (6) and (5) allows us to say that

quantum correlations beyond the entanglement itself can survive to a pertubative environment which the action is similar to a partial local measurement. According to that, a QI radar application could be interesting to develop in a frequency interval where the photon-medium interaction is quite low.

## IV. Conclusion

In this paper, we performed a study of the quantum information evolution in function of a mathematical parameter $\Gamma$ depicting the interaction strength between a photon i.e. qubit and its near environment. We worked in a Quantum Illumination Radar scheme similar to Lloyd's approach [3]. We modeled the propagation medium using a partial local measurement operator that changes the quantum state AS taking in consideration that the interaction makes evolve the system from a entangled state to a mixed state. With this propagation model, we showed that one part of initial quantum information can survive despite the entanglement loss due to the decoherence produced by the propagation environment. The remaining quantum information could be exploited in a remote detection so in a Quantum Illumination process. The use of multiple emissions of photon pairs is needed to reduce the error probability to discriminate both quantum states $\hat{\rho}_{AS}^{(e)}$ and $\hat{\rho}_{AS}^{(sep)}$.

The work presented in this paper is a preliminary study of the propagation medium influence in a QI radar scheme. Until now, we did not consider the thermal noise in the process. That can represent a relevant continuity for this work.

## Acknowledgment

The research work is carried out with the support of the AID-DGA (Defense Innovation Agency - French Ministry of the Armed Forces), hence the authors hereby express their gratitude and thanks to AID.

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