A global optimization approach to H infinity synthesis with parametric uncertainties applied to AUV control

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Abstract: Given a Linear Time Invariant (LTI) system with parametric uncertainties, we propose a new method to synthesize a structured controller with H_∞ constraints. Our approach is based on global optimization. The problem is formulated as a min-max optimization problem. A new version of a global optimization algorithm based on interval arithmetic is implemented to solve this kind of problems. To validate our approach, an example of Autonomous Underwater Vehicle (AUV) regulation to synthesize a PID controller with two H_∞ constraints and two parametric uncertainties is given.

Keywords: H_∞ constraints, Structured synthesis, Robust Control, Global optimization, Parametric uncertainties, Interval Analysis, Linear System

1. INTRODUCTION

When a dynamic model of a physical phenomenon is constructed, some of its parameters are known with uncertainties. These uncertainties can come from simplifications of physical laws, or proceed from identification from real data, etc. To find a robust controller which stabilizes the real system, it is interesting to take into account these uncertainties in the control synthesis problem. Therefore, we propose, in this paper, a new methodology to synthesis a controller which respects robust criteria for all possible values of the uncertain parameters. A large field of control synthesis is devoted to stabilization subject to H_∞ constraints. Many approaches propose to tune a full order controller. In our approach, a reduced order controller is proposed with a given structure as first proposed in 2006 by Burke et al. (2006) and Apkarian and Noll (2006). The main objectives of a structured controller is to make easy the implementation or the interpolation in the case of a Linear Parameter Varying (LPV) systems.

Early methods, which deals with uncertainties, were based on the Structured Singular Value (SSV) approach. But, these methods impose a rank constraint on the controller (Zhou and Doyle (1998)). However, recent work based on the SSV proposes the robust synthesis of structured controller (Apkarian (2011)). Other approaches consider the uncertainties to be ellipsoid parametric uncertainties, in order to formulate the synthesis problem like a Linear Matrix Inequalities (LMI) problem (Peaucelle and Arzelier (2005); Barenthin and Hjalmarsson (2008); Sadeghzadeh et al. (2011)). Uncertainties can also be represented by intervals. In this case, set-membership approaches using interval arithmetic have been considered (Jaulin and Walter (1996); Malan et al. (1997, 1994)). Another approach of the problem considers the worst case of all the uncertainties. Instead of considering all uncertainties, only the value of uncertainty that maximize the criteria are considered (Apkarian and Noll (2016); Bianco and Piazzi (1998)).

In this paper, we consider a synthesis problem subject to robust stability constraints and H_∞ constraints based on our previous work (Monnet et al. (2015)). We propose a worst-case approach to deal with the uncertainties, and formulate the synthesis problem like a min-max optimization problem. A global optimization method is used to provide a guaranteed enclosure of the minimum.

The paper is organized as follows. Section 2 introduces H_∞ synthesis issue and proposes a min-max formulation of the controller synthesis problem with parametric uncertainties. Section 3 presents the robust stabilization problem. Section 4 introduces a global optimization method based

[⋆] This work is supported by the city of Brest (Brest Metropole) and the DGA (French Defense Procurement Agency).

on interval arithmetic and formulates the control synthesis problem subject to stability constraint. Section 5 validates our original approach on a control problem dedicated to the regulation of an underwater robot in comparison with a previous work (Yang et al. (2015)).

2. H_∞ CONSTRAINTS WITH PARAMETRIC UNCERTAINTIES

2.1 H_∞ synthesis

To control a Linear Time Invariant (LTI) system G, H_∞ synthesis (Zhou and Doyle (1998)) computes a LTI system K (named the controller) such that the system G, interconnected with K by a feedback loop, offers the wanted behavior: small tracking error, perturbation rejection, and any performances an engineer could imagine, etc. To do so, an augmented system P is built from G and the weighting functions, which penalize the non-desired behaviors. The inputs of P are noted: w, the vector of exogenous inputs (reference signals, noises, perturbations on control, etc.); u, the control vector. The outputs of P are noted: z, the vector of performance outputs (tracking error, control signal, etc.); y, the measurement vector. Figure 1 summarizes the notations and shows the interconnection between the plant P and the controller K.

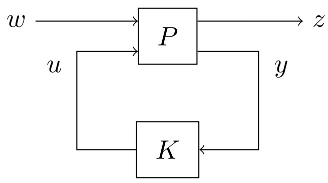

Fig. 1. Interconnection of the augmented system P with the controller K.

Let $F(P, K)$ be the linear fractional transform (LFT) of P with K, which describes the closed-loop system represented on Figure 1. $F(P, K)$ can be described by a matrix of transfer functions that maps the inputs w to the outputs z,

$$F(P,K) = \begin{pmatrix} T_{w\to z_1}(s) \\ \vdots \\ T_{w\to z_q}(s) \end{pmatrix}$$

where $s = j\omega$ denotes the Laplace variable, j denotes the imaginary unit, $\omega \in \mathbb{R}$ denotes the pulsation, and $T_{w\to z_i}(s) = (T_{w_1\to z_i}(s), \ldots, T_{w_n\to z_i}(s))$ is a row vector of transfer functions which maps w to z_i. The principle of H_∞ synthesis is to find K that minimizes the sensitivity of every performance outputs z_i from the inputs w. This sensitivity can be quantified by the H_∞ norm. The H_∞ synthesis problem is formulated as a Constraint Satisfaction Problem (CSP):

$$\text{find } K \text{ such that } \|T_{w\to z_i}\|_\infty \le 1, \quad \forall i \in \{1, \ldots, q\}. \quad (1)$$

We recall that the H_∞ norm $\|.\|_\infty$ is defined by:

$$\|T_{w\to z_i}\|_\infty = \sup_{w\neq 0} \frac{\|z_i(j\omega)\|_2}{\|w(j\omega)\|_2}$$

The H_∞ control synthesis problem can also be formulated as a optimization problem:

$$K = \underset{K}{argmin} \left(\max_{i\in\{1,\ldots,q\}} \|T_{w\to z_i}\|_\infty \right), \quad (2)$$

If the minimum value of Problem (2) is lower than 1, the solution of Problem (2) is a solution of Problem (1).

2.2 Structured H_∞ synthesis

Problem (2) implies to search a controller in the space of all LTI systems. However, it is interesting to fix the structure of the controller K. Such design constraint is tremendously difficult to take into account in an optimization based control design because it renders the set of admissible controllers nonconvex. This structured controller is defined by variables, noted $k \in \mathbb{R}^{n_k}$ where n_k is the number of parameters to tune. Hence, to find a solution of Problem (2), we limit the search for all the controller $K(k)$ with $k \in \mathbb{K} \subset \mathbb{R}^{n_k}$. For example, a Proportional Integral Derivative (PID), the variable $k \in \mathbb{K}$ is the coefficients $k = (k_i, k_p, k_d) \in \mathbb{R}^3$, with $K = k_i/s + k_p + k_d s$. Problem (2) can be reformulated as a structured H_∞ synthesis problem:

$$\min_{k\in\mathbb{K}} \left(\max_{i\in\{1,\ldots,q\}} \|T_{w\to z_i}(k)\|_\infty \right). \quad (3)$$

Note that each $T_{w\to z_i}$ depends on k.

2.3 H_∞ synthesis with parametric uncertainties

Let us consider that $G(p)$ depends on uncertain parameters $p \in \mathbb{P}$, where $\mathbb{P}$ is the set of all possible values of these uncertainties. The H_∞ synthesis problem with parametric uncertainties can be expressed as the minimization of the worst case over $\mathbb{P}$ of the H_∞ constraints. To minimize $\|T_{w\to z_i}\|_\infty$ for all $p \in \mathbb{P}$, we minimize the maximum: $\sup_{p\in\mathbb{P}} \|T_{w\to z_i}(p, k, j\omega)\|_\infty$. Therefore, the structured H_∞ synthesis problems with parametric uncertainties can be expressed as the following min-max problem:

$$\min_{k\in\mathbb{K}} \left(\sup_{p\in\mathbb{P}} \big(\max_{i\in\{1,\ldots,q\}} \|T_{w\to z_i}(p, k, j\omega)\|_\infty \big) \right). \quad (4)$$

Remark 1. If the minimum of Problem (4) is lower than 1, the solution of Problem (4) is a solution to the following CSP:

$$\text{find } k \text{ such that } \begin{cases} \|T_{w\to z_i}(p, k)\|_\infty \le 1, \\ \forall i \in \{1, \ldots, q\}, \forall p \in \mathbb{P}. \end{cases} \quad (5)$$

3. ROBUST STABILIZATION

The H_∞ synthesis guarantees performances of the closed-loop system $F(G, K)$. But, the controller must also stabilize the closed-loop system. Thus, the solution of Problem (4) does not necessarily stabilize $G(p)$. Therefore, we must ensure that $K(k)$ stabilizes the closed-loop system for all $p \in \mathbb{P}$. This is a problem of robust stabilization:

$$\forall p \in \mathbb{P},\ K(k) \text{ stabilizes } F(G(p), K(k)),$$

This problem was first solved by Kharitonov (1978), and was approached later with Interval Analysis by Jaulin and Walter (1996) using the Routh-Hurwitz criterion.

The optimization problem, we consider, is the minimization of H_∞ objective function subject to robust stabilization constraints with parametric uncertainties:

$$\min_{k\in\mathbb{K}}\left(\sup_{p\in\mathbb{P}}\big(\max_{i\in\{1,\dots,q\}}\|T_{w\to z_i}(p,k,j\omega)\|_\infty\big)\right)$$
$$\text{subject to } \forall p\in\mathbb{P},\ K(k) \text{ stabilizes } F(G(p),K(k)).$$

4. GLOBAL OPTIMIZATION APPROACH

In this section, an Interval Branch and Bound Algorithm (IBBA) is presented (Ninin et al. (2014)). This algorithm is a deterministic global optimization algorithm (Kearfott (1992)). It provides a guaranteed enclosure of the optimum of a non-convex problem. To use this algorithm, we reformulate our problem as a constrained non-convex problem, with an H_∞ objective function and robust-stability constraints.

4.1 Interval Branch and Bound algorithm

In order to present the algorithm, we first define intervals.

Definition 1. An interval is a closed connected subset of $\mathbb{R}$ (Moore et al. (2009)), described by its endpoints $\underline{x}$ and $\overline{x}$:

$$\boldsymbol{x}=[\underline{x},\overline{x}]=\{x\mid \underline{x}\le x\le\overline{x}\},$$
$$\text{with } \underline{x}\in\mathbb{R}\cup\{-\infty\} \text{ and } \overline{x}\in\mathbb{R}\cup\{+\infty\}$$

The set of intervals is denoted by $\mathbb{IR}$ and the set of n-dimensional interval vectors, also called boxes, is denoted by $\mathbb{IR}^n$.

Consider a constrained optimization problem:

$$\begin{cases}\min\limits_{x\in\mathbb{X}} f(x)\\ \text{s.t. } \ g_m(x)\le 0,\ \forall m.\end{cases}\tag{6}$$

where $f:\mathbb{R}^n\mapsto\mathbb{R}$, $g_m:\mathbb{R}^n\mapsto\mathbb{R}$, and $\mathbb{X}\subseteq\mathbb{IR}^n$. An Interval Branch and Bound Algorithm (Ninin et al. (2014)) provides a feasible solution $\tilde{x}$ and a guaranteed enclosure of the global minimum $\boldsymbol{\mu}$ of f over $\mathbb{X}$. The principle of the algorithm is a branch-and-bound algorithm based on Interval Analysis. Bounds of the objective function are computed on $\mathbb{X}$. If these bounds are too wide, the algorithm splits $\mathbb{X}$ into smaller boxes, and provide smaller enclosures of f over each boxes. Figure 2 illustrates the principle of the algorithm. The bounds of f over each $\mathbb{X}_i$ can be over-approximated using interval analysis (Jaulin et al. (2001)).

Definition 2. An inclusion function $[f]$ of f is defined from $\mathbb{IR}^m$ into $\mathbb{IR}$ and respects the following property:

$$f(\mathbb{X}_i)=\{f(x),\,x\in\mathbb{X}_i\}\subseteq[f](\mathbb{X}_i).$$

IBBA deletes $\mathbb{X}_i$ when it is proved that the global minimum is not inside. For example, if $\tilde{x}$ is a feasible solution,

$$\underline{[f](\mathbb{X}_i)}>f(\tilde{x})\implies\forall x_i\in\mathbb{X}_i, f(x_i)>f(\tilde{x})$$
$$\implies x^*\notin\mathbb{X}_i.$$

To improve the converge of IBBA, constraint propagation techniques and pruning techniques are included to reduce the number or the size of the sub-boxes $\mathbb{X}_i$ (Ninin et al. (2014)). For example, if $\underline{[g_1](\mathbb{X}_i)}>0$, $\mathbb{X}_i$ is deleted because:

$$\forall x\in\mathbb{X}_i,\ g(x)>0\implies x^*\notin\mathbb{X}_i.$$

Indeed, IBBA converges to a small sub-box $\boldsymbol{x}^*$ of $\mathbb{X}$ which contains x^*. Moreover, considering the remaining boxes, IBBA provides a guaranteed enclosure of the global minimum:

$$f(x^*)\in\boldsymbol{\mu}=\left[\min_i\underline{[f](\mathbb{X}_i)}\ ,\ \max_i\overline{[f](\mathbb{X}_i)}\right].$$

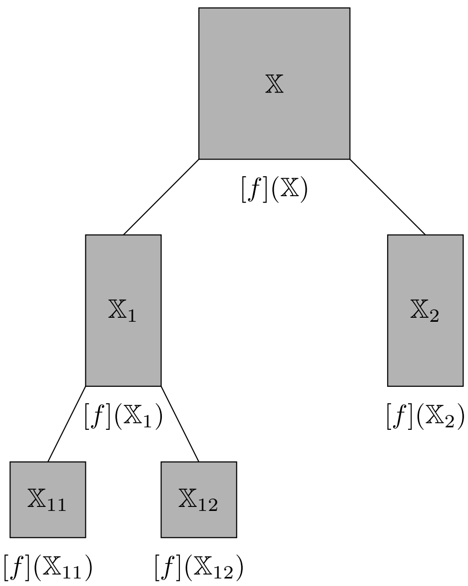

Fig. 2. Interval Branch and Bound Algorithm.

Consider now the constrained min-max problem:

$$\begin{cases}\min\limits_{x\in\mathbb{X}}\ \sup\limits_{y\in\mathbb{Y}} f(x,y)\\ \text{s.t. } \ g_m(x,y)\le 0,\ \forall y\in\mathbb{Y},\forall m.\end{cases}\tag{7}$$

Problem (7) can be solved using two IBBA, see Monnet et al. (2016b). A first IBBA splits $\mathbb{X}$ into subsets $\mathbb{X}_i$, and a second IBBA is used to compute an enclosure of $f(\mathbb{X}_i,\mathbb{Y})$ by solving the maximization problem:

$$\sup_{y\in\mathbb{Y}} f(\mathbb{X}_i,y).$$

Note that $f(\mathbb{X}_i,.)$ is a family of functions, which means that IBBA will converge to

$$\mathbb{Y}_i=\{y\in\mathbb{Y}\mid\exists x\in\mathbb{X}_i,\ y \text{ maximizes } f(x,y)\}$$

and provide an enclosure $\boldsymbol{\mu}_i$ of all maximums,

$$\{\sup f(x,y),\ x\in\mathbb{X}_i\}\subseteq\boldsymbol{\mu}_i=[\underline{\mu}_i,\overline{\mu}_i]$$

4.2 Constrained min-max formulation

In order to solve Problem (4) with a global optimization approach, we propose to minimize the following problem:

$$\min_{k\in\mathbb{K}}\left[\sup_{p\in\mathbb{P}}\left(\max_{i\in\{1,\dots,q\}}\|T_{w\to z_i}(p,k,j\omega)\|_\infty\right)\right].\tag{8}$$

In Monnet et al. (2016a), we have the following result:

$$\|T_{w\to z_i}\|_\infty=\sup_{\omega>0}\sqrt{\sum_{l=1}^{n}|T_{w_l\to z_i}(j\omega)|^2}$$

with $T_{w\to z_i}(j\omega)$ a row vector. Therefore, Problem (8) can be expressed as follows:

$$\min_{k\in\mathbb{K}}\left[\sup_{\omega>0,p\in\mathbb{P}}\left(\max_{i\in\{1,\dots,q\}}\sqrt{\sum_{l=1}^{n}|T_{w_l\to z_i}(p,k,j\omega)|^2}\right)\right].$$

Consider

$$f(p,k,\omega) = \max_{i\in\{1,\dots,q\}} \sqrt{\sum_{l=1}^{n} |T_{w_l \to z_i}(p,k,j\omega)|^2},$$

it is possible to define $[f]$ an inclusion function of f (Monnet et al. (2016b)).
The robust stability constraint can be expressed as a system of inequalities using the Routh-Hurwitz criterion,

$$\forall p \in \mathbb{P}, \begin{cases} R_1(k,p) \le 0 \\ \vdots \\ R_r(k,p) \le 0 \end{cases}$$

where R_i are scalar valued functions, for which inclusion functions $[R_i]$ can also be defined (Jaulin and Walter (1996)).
Therefore, the structured H_∞ synthesis problem with parametric uncertainties and robust stabilization constraints can be formulated as follows:

$$\begin{cases} \min\limits_{k\in\mathbb{K}} & \sup\limits_{\omega>0, p\in\mathbb{P}} f(k,p,j\omega) \\ & R_1(k,p) \le 0, \forall p \in \mathbb{P} \\ \text{subject to} & \vdots \\ & R_r(k,p) \le 0, \forall p \in \mathbb{P} \end{cases} \tag{9}$$

Problem (9) is solved with a new version of IBBA presented in Section 4.1.

Remark 2. Solving Problem (9) with IBBA implies that $\mathbb{P}$ is a box. It means that the uncertainties of the value of the parameters are represented by an interval. This representation is generally suited to describe uncertainties.

5. EXAMPLE

We propose to illustrate our global optimization based controller synthesis approach with the example of the control of a cubic Autonomous Underwater Vehicle (AUV), described by Yang et al. (2015, 2014). This is a classical control problem in marine robotics that have been investigated in recent works (Roche et al. (2011),Maalouf et al. (2015)). The proposed problem is the regulation of the yaw angle of the AUV with parametric uncertainties and H_∞ constraints. Indeed, the modelling of a underwater system is a complex task due to the modelling of the environment and the hydrodynamic phenomenons.

The regulation scheme is represented in Figure 3.

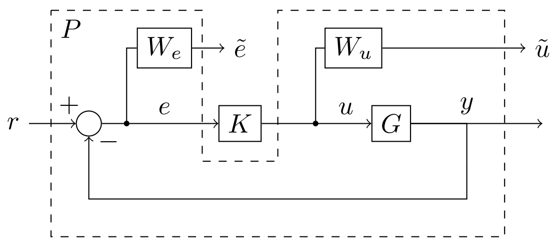

Fig. 3. Regulation scheme with weighting functions.

The signals $\tilde{e}$ and $\tilde{u}$ represent the weighted outputs, and r the reference signal. The augmented system mentioned in Section 2 is delimited by dotted lines.
The yaw dynamic of the AUV is described by the transfer function:

$$G(p,s) = \frac{1}{p_1 s^2 + p_2 s},$$

with

$$p = (p_1, p_2)^T,$$

where $p \in \mathbb{R}^2$ is the vector of uncertain parameters, $p \in \mathbb{P} = [0.30, 0.69] \times [1.26, 2.34]$.
We choose to control the AUV yaw angle with a filtered Proportional Integral Derivative (PID) controller,

$$K(k,s) = k_p + \frac{k_i}{s} + \frac{k_d s}{1+s},$$

with

$$k = (k_p, k_i, k_d)^T.$$

The variables of the controller are searched in:

$$\mathbb{K} = [-5,5] \times [-5,5] \times [-5,5].$$

Two H_∞ constraints are defined on the sensitivity transfer $S(p,k,s) = (I + G(p,s)K(k,s))^{-1}$ and the transfer from the reference signal to the control signal $K(k,s)S(p,k,s)$:

$$\|W_e(s)S(p,k,s)\|_\infty \le 1,$$
$$\|W_u(s)K(k,s)S(p,k,s)\|_\infty \le 1.$$

$W_e(s)$ and $W_u(s)$ are the weighting functions defined by

$$W_e(s) = 0.5\frac{s+0.92}{s+0.0046},$$
$$W_u(s) = 0.01.$$

The robust synthesis problem consists in finding k such that the H_∞ constraints are respected for all possible values of p, and can be formulated as the minimization of the worst case with respect to the uncertainties as explained in Section 2:

$$\begin{cases} \min\limits_{k\in\mathbb{K}} \left[\sup\limits_{p\in\mathbb{P}} \Bigg(\max\left(\|W_e(s)S(p,k,s)\|_\infty \right., \right. \\ \qquad \left. \left. \|W_u(s)K(k,s)S(p,k,s)\|_\infty\right) \Bigg) \right] \\ \text{s.t. } K(k,s) \text{ robustly stabilizes the closed loop.} \end{cases} \tag{10}$$

Problem (10) is reformulated as explained in Section 4 and solve with our new version of IBBA. We limit the frequency range to $\omega \in [10^{-3}, 10^3]$.

We obtain:

$$\max\left(\|W_e(s)S(p,k^*,s)\|_\infty,\ \|W_u(s)K(k^*,s)S(p,k^*,s)\|_\infty\right) \in \boldsymbol{\mu} = [0.45, 0.55],$$

where k^* is the solution to Problem (10). The best solution found is:

$$K(\tilde{k},s) = 1.471 + \frac{0.103}{s} + \frac{1.471s}{1+s}.$$

Our solution $\tilde{k}$ has been found such that $\overline{\mu} < 1$ thus, we guarantee that the H_∞ constraints are respected for any value of p_1 and p_2 in $\mathbb{P}$. However, if $\underline{\mu}$ were greater than 1, it would prove that there exist at least one value of p (the worst case) such neither of the constraints is respected $\forall k \in \mathbb{K}$. Indeed, if $\underline{\mu} > 1$, Problem (5) is not feasible. In the same way, if $1 \in \boldsymbol{\mu}$, we cannot prove anything.
Figure 4 shows the Bode diagrams of the objective channels S and KS for 20 values of the uncertainties p. It shows that all the frequency responses respects the objectives. We remark that for every p, S and KS are under the inverse of the weighting functions W_e^{-1} and W_u^{-1}, which is consistent with the fact that:

$$\|W_e S(\tilde{k})\|_\infty < 1$$

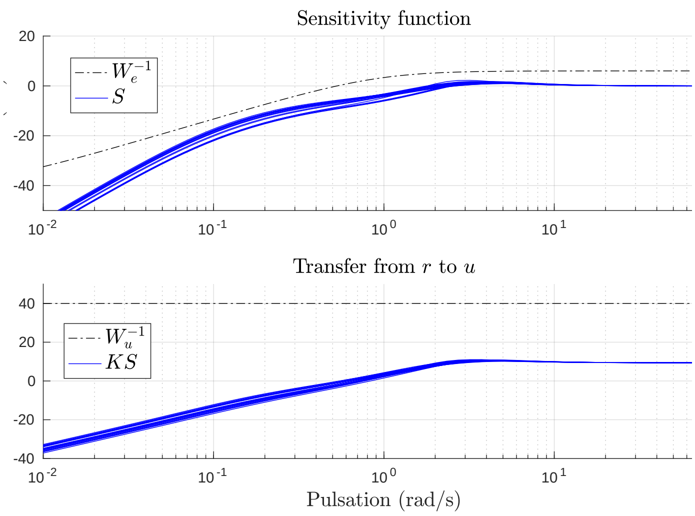

Fig. 4. Frequency template for objective channels.

and

$$\|W_u K(\tilde{k}) S(\tilde{k})\|_\infty < 1.$$

Figure 5 shows the poles location of the closed-loop transfer function $T(p, \tilde{k}, s) = G(p, s)K(\tilde{k}, s)S(p, \tilde{k}, s)$ for several values of p in $\mathbb{P}$. The poles are located on the left half plan

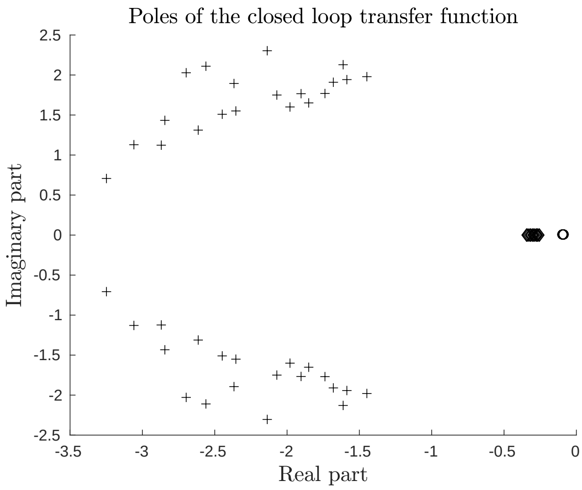

Fig. 5. Poles of $T(p, \tilde{k}, s)$ for 20 values of p.

of the complex plan, which illustrates the stability of the closed loop. Note that the poles locus shows that only the very stable poles are moving changing p, which shows the robustness of the proposed controller.

The step response performances of the closed loop are shown on Figure 6. A reference step is imposed at $t = 10s$ and a step disturbance is imposed at $t = 50s$. And it can be seen that all the regulation objectives are fulfilled.

We compare our approach with the structured H_∞ synthesis method implemented in the `Systune` toolbox of `Matlab`. Since this method cannot take uncertainties into account, we propose to synthesize a controller for the

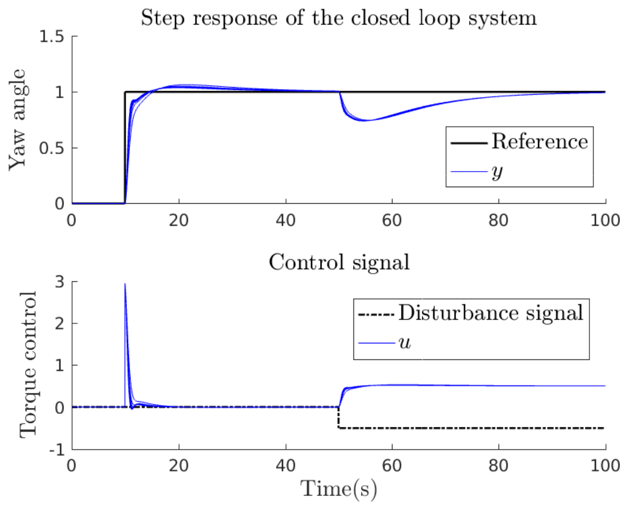

Fig. 6. Time responses for a step response and a step disturbance. The yaw response y and the command input u.

nominal plant $G_n = G(p_n)$, with $p_n = (0.7, 1.8)$. With `Systune`, we obtain the controller

$$K_{st} = K(k_{st}, s),\ k_{st} = (1.372, 0, 0.74)^T. \tag{11}$$

which provides for the nominal plant

$$\begin{aligned}\max\big(\ & \|W_e(s)S(p_n, k_{st}, s)\|_\infty, \\ & \|W_u(s)K_{st}S(p_n, k_{st}, s)\|_\infty\big) = 0.6.\end{aligned}$$

With our global optimization approach, we can perform a robustness analysis for K_{st}. Indeed, $\max_i([R_i](k_{st}, \mathbb{P}) =$ which proves the robust stability of the closed loop, and by solving the "sup" part of Problem (10) at k_{st}, we obtain

$$\begin{aligned}\max\big(\ & \|W_e(s)S(p_{st}, k_{st}, s)\|_\infty, \\ & \|W_u(s)K_{st}S(p_{st}, k_{st}, s)\|_\infty\big) = 0.78.\end{aligned}$$

where p_{st} denotes the worst case over $\mathbb{P}$ at k_{st}. As a consequence, our structured robust synthesis approach performs better than a non-robust synthesis method on this example.

This simple example shows that the proposed approach can lead to an efficient methodology for PID tuning under H_∞ constraints.

6. CONCLUSION

A new method to synthesize a controller with H_∞ constraints, stability constraints and parametric uncertainties is presented. A worst-case approach of this problem is formulated as a constrained min-max problem. A global optimization algorithm provides an upper and a lower bound on the minimum of the H_∞ constraint. These bounds determine which performances can be achieved with a given parametrized controller, and prove in a guaranteed way whether there exists a controller such that the H_∞ constraints are respected for all possible values of uncertainties or not. Moreover, the stability of the closed-loop transfer function is ensured for all possible value of the uncertainties. This paper is also dedicated to the comparison between full order synthesis and structured synthesis with a simple robotic example; it is based on the previous work from (Yang et al. (2015)) and it shows that we can compute a PID with similar performances as the full order controller obtained by classical H_∞ synthesis. This work is the fisrt step for robust multiobjective control synthesis based on previous approaches (Abbas-Turki et al. (2006); Arzelier et al. (2006)) and applied on various application such that aerospace launcher (Clement et al. (2005)), autonomous sailboats (Clement (2013)) and others.

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