

A column generation approach for coordination and control of multiple UAVs

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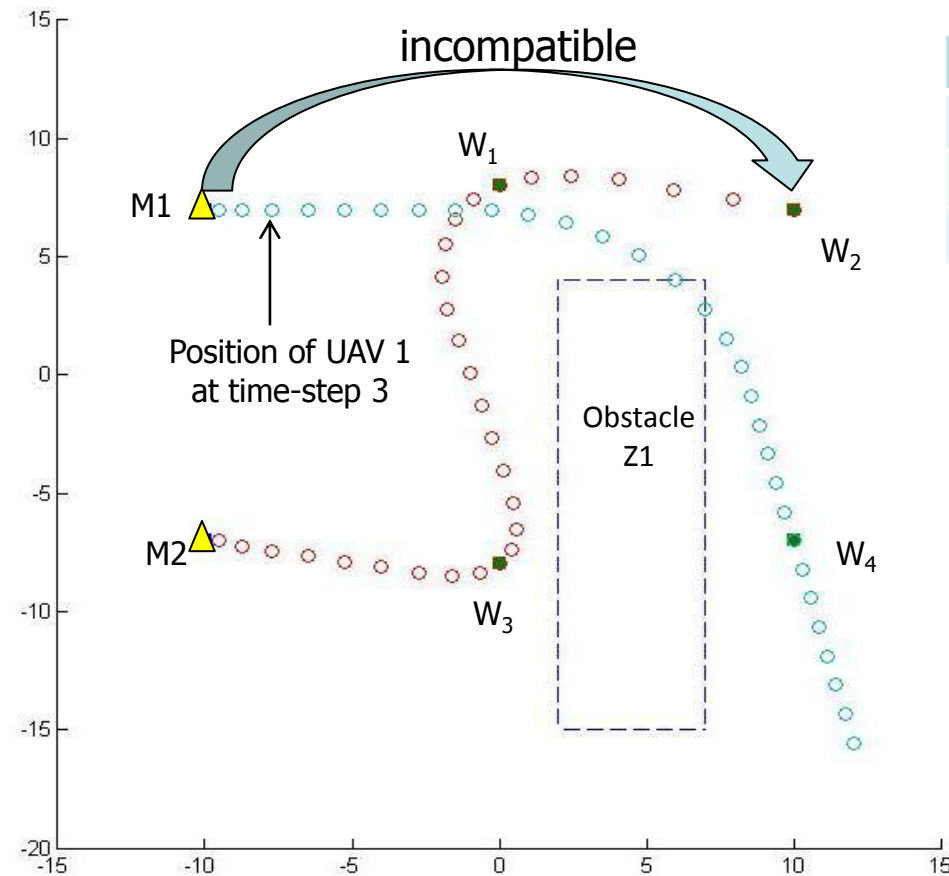
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Plan

- ❑ Introduction
- ❑ Formulation of problem
- ❑ Column generation method
- ❑ Branching
- ❑ Numerical experiments
- ❑ Conclusion and perspectives

Introduction



■ M1-M2	UAV
■ W1-W4	Waypoint
○	Position of UAV at each time-step
□ Z1	Obstacle

Bi-level problem:

the upper-level : task assignment problem

[capability constraints]

the lower-level : constructs the detailed trajectory of UAVs

[dynamic system]

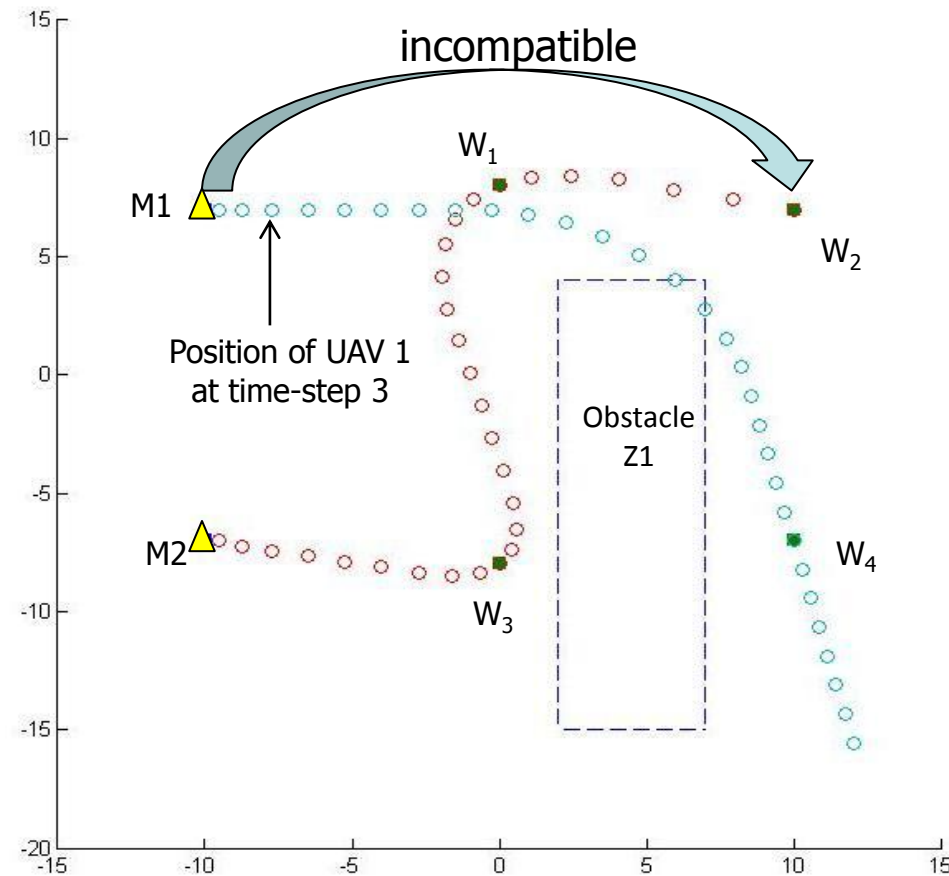
[avoidance the obstacles]

[dependency constraints]

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Formulation (1/5)



N_v : Number of vehicles (UAVs)

index : p

N_w : Number of navigation points

index : i , (W_{i1}, W_{i2}) : position

N_z : Number of no-fly-zones

index : j

N_T : time steps are used for planning

index : t

UAV: point mass moving in two dimensions with limited speed and turning rate

→ state of UAV : (x, y, \dot{x}, \dot{y})

Formulation (2/5)

The model of aircraft dynamics :

$$\dot{x}_{tp} \sin \left(\frac{2\pi h}{N_c} \right) + \dot{y}_{tp} \cos \left(\frac{2\pi h}{N_c} \right) \leq v_{max_p},$$
$$\forall t = 1, \dots, N_T, \forall p = 1, \dots, N_V, \forall h = 1, \dots, N_C, \quad (1)$$

$$f_{x_{tp}} \sin \left(\frac{2\pi h}{N_c} \right) + f_{y_{tp}} \cos \left(\frac{2\pi h}{N_c} \right) \leq f_p,$$
$$\forall t = 0, \dots, N_T - 1, \forall p = 1, \dots, N_V, \forall h = 1, \dots, N_C. \quad (2)$$

$$\omega_p = \frac{f_p}{m_p \cdot v_{max_p}}. \quad (3)$$

$$s_{(t+1)p} = \mathbf{A}s_{tp} + \mathbf{B}f_{tp},$$
$$\forall t = 0, \dots, N_T - 1, \forall p = 1, \dots, N_V. \quad (4)$$

Formulation (3/5)

Avoid a rectangular obstacle:

$$\begin{aligned}x_{tp} - Z_{j3} &\geq -Rc_{jpt1}, \\Z_{j1} - x_{tp} &\geq -Rc_{jpt2}, \\y_{tp} - Z_{j4} &\geq -Rc_{jpt3}, \\Z_{j2} - x_{tp} &\geq -Rc_{jpt4}, \\ \sum_{z=1}^4 c_{jptz} &\leq 3, \\ \forall t = 1, \dots, N_T, \forall p = 1, \dots, N_V, \forall j = 1, \dots, N_Z,\end{aligned} \tag{5}$$

Visit a waypoint:

$$\begin{aligned}x_{tp} - W_{i1} &\leq R(1 - b_{ipt}), \\x_{tp} - W_{i1} &\geq -R(1 - b_{ipt}), \\y_{tp} - W_{i2} &\leq R(1 - b_{ipt}), \\y_{tp} - W_{i2} &\geq -R(1 - b_{ipt}), \\ \forall t = 1, \dots, N_T, \forall p = 1, \dots, N_V, \forall i = 1, \dots, N_W.\end{aligned} \tag{6}$$

Formulation (4/5)

Assignment and Time dependency:

Time dependency:

- Each couple :

$$D_k = (i_k, j_k), t_{D_k}$$

The waypoint i_k must be visited after the waypoint j_k a duration of t_{D_k} time units.

$$\sum_{t=1}^{N_T} \sum_{p=1}^{N_V} K_{pi} b_{ipt} \leq 1, \forall i \in N_W \setminus \mathbf{D},$$

$$\sum_{t=1}^{N_T} \sum_{p=1}^{N_V} K_{pi} b_{ipt} = 1, \forall i \in \mathbf{D},$$

(7)

$$\sum_{t=1}^{N_T} \sum_{p=1}^{N_V} t b_{i_k, p, t} - \sum_{t=1}^{N_T} \sum_{p=1}^{N_V} t b_{j_k, p, t} \geq t_{D_k}, \forall k = 1, \dots, N_D.$$

(8)

Formulation (5/5)

Resources:

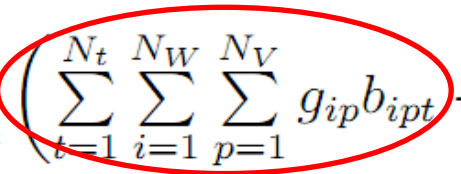
$$\sum_{t=0}^{N_T-1} (|f_{tp}^x| + |f_{tp}^y|) \leq F_p, \forall p = 1, \dots, N_V \quad (9)$$

The flight completion time:

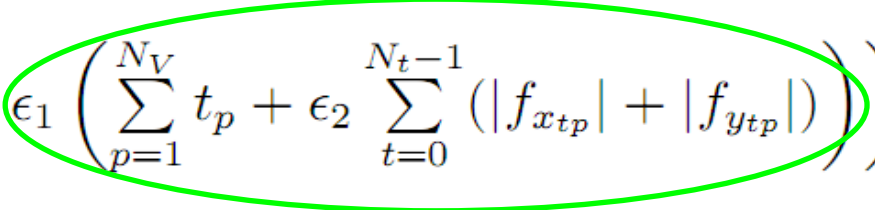
$$t_p \geq \sum_{t=1}^{N_T} tb_{ipt}, \forall p = 1, \dots, N_V, \forall i = 1, \dots, N_W. \quad (10)$$

The optimization problem:

$$\left\{ \begin{array}{l} \max_{s,f,b,c} \left(\sum_{t=1}^{N_t} \sum_{i=1}^{N_W} \sum_{p=1}^{N_V} g_{ip} b_{ipt} - \epsilon_1 \left(\sum_{p=1}^{N_V} t_p + \epsilon_2 \sum_{t=0}^{N_t-1} (|f_{x_{tp}}| + |f_{y_{tp}}|) \right) \right) \\ \text{subject to : (1) - (10)} \end{array} \right. \quad (11)$$



gain



cost

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Column generation (1/11)

- **Column generation**: a prominent method to cope with linear programming (integer or mixed) with a huge number of variables
- In context of column generation, **branch-and-price** is an indispensable procedure for globally solving a problem. Several specialized branch-and-price algorithms : generalized assignment problem, vehicle routing problems, scheduling problems, etc.

[2] Dantzig GB, Wolfe P. *Decomposition principle for linear programs*. Operations Research 1960;8:101-111.

[3] Desaulniers G, Desrosiers J, Solomon MM (Eds.). *Column Generation*, Springer US (2005).

[4] Lübbecke M, Desrosiers J. *Selected Topics in Column Generation*. Operations Research 2005;53(6):1007-1023.

Column generation (2/11)

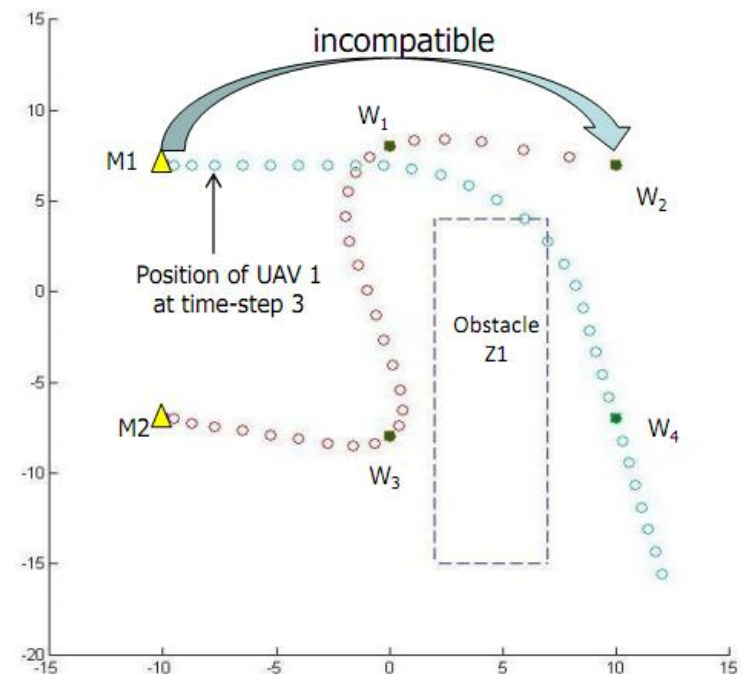
Feasible trajectory :

A feasible trajectory of a vehicle p is a trajectory starting from its departure, satisfying all moving constraints and visiting at least one waypoint i .

Ω_p : the set of all feasible trajectories for the vehicle p .

Ω : the set of all feasible trajectories

$$\Omega = \bigcup_{p \in N_V} \Omega_p .$$



Column generation (3/11)

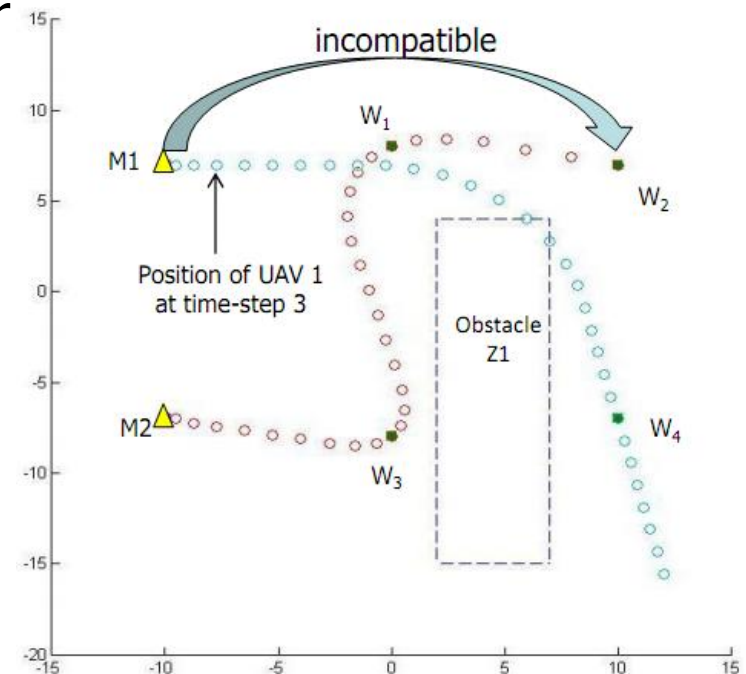
Let $r = (r_1, r_2, \dots, r_{N_T}) \in \Omega_p \subset \Omega$ be a feasible trajectory, where r_1, \dots, r_{N_T} are waypoints visited by UAV p. The performance of this trajectory, denoted $g(r)$, is calculated:

$$g(r) = \sum_{i \in N_W \cap r} g_{i,p} - \epsilon_1 (t_r^p + \epsilon_2 f_r^p).$$

$\sum_{i \in N_W \cap r} g_{i,p}$: the gain of trajectory r

t_r^p : the moment when UAV p visits the last waypoint of the trajectory r

f_r^p : the total force used through the trajectory r



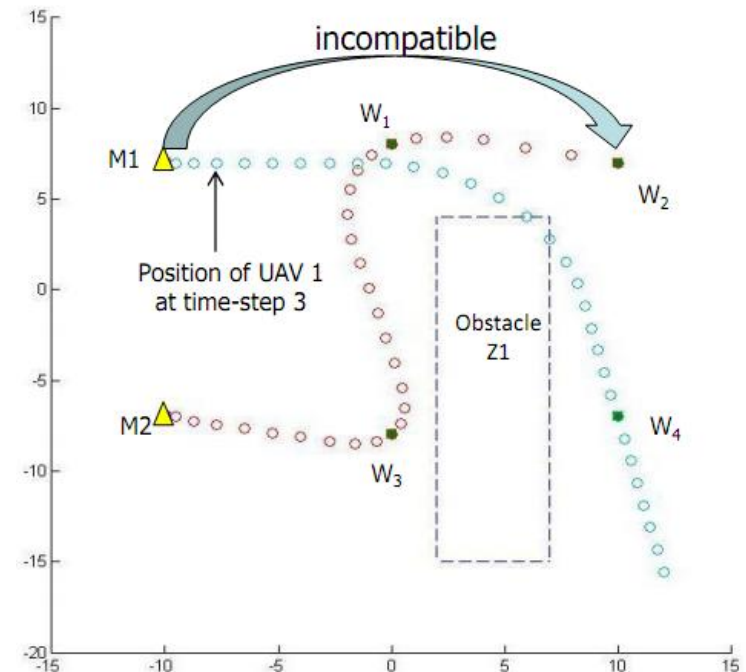
Column generation (4/11)

We define, for each waypoint $i \in N_W$, the parameter of visitation a_{ri} , by :

$a_{ri} = 1$ if trajectory r passes waypoint i ,
 $a_{ri} = 0$ otherwise.

$t_r(i)$: the time-step for
visiting waypoint i

$\Omega_p(i)$: the set of all feasible
trajectories of UAV p
visiting waypoint i



Column generation (5/11)

Problem (11) can be reformulated:

$$\left\{ \begin{array}{l} \max \sum_{p=1}^{N_V} \sum_{r \in \Omega_p} g(r) \cdot \theta_r \\ \text{s.t.} \sum_{p=1}^{N_V} \sum_{r \in \Omega_p} a_{ri} \theta_r \leq 1, \forall i = 1, \dots, N_W, \\ \sum_{r \in \Omega_p} \theta_r \leq 1, \forall p = 1, \dots, N_V, \\ \sum_{p=1}^{N_V} \sum_{r \in \Omega_p(i_k)} t_r(i_k) \theta_r - \sum_{p=1}^{N_V} \sum_{r \in \Omega_p(j_k)} t_r(j_k) \theta_r \geq t_{D_k}, \forall k = 1, \dots, N_D, \\ \sum_{p=1}^{N_V} \sum_{r \in \Omega_p(i_k)} \theta_r = 1, \forall k = 1, \dots, N_D, \\ \sum_{p=1}^{N_V} \sum_{r \in \Omega_p(j_k)} \theta_r = 1, \forall k = 1, \dots, N_D, \\ \theta_r \in \{0, 1\}, \forall r \in \Omega. \end{array} \right. \quad (12)$$

The variable $\theta_r \in \{0, 1\}$ is a decision variable which describes whether a trajectory r is chosen or not.

Column generation (6/11)

Due to the first constraint, we can replace

$$\theta_r \in \mathbf{N}, \forall r \in \Omega. \quad \leftarrow \quad \cancel{\theta_r \in \{0, 1\}, \forall r \in \Omega}$$

The linear relaxation of problem (12), i.e.,

$$\theta_r \geq 0, \forall r \in \Omega$$

is called Master Problem (MP).

Column generation (7/11)

- Given $\Omega_k^1 \subset \Omega_k, k \in K$ and $\Omega^1 = \bigcup_{k \in K} \Omega_k^1$. We consider the restricted Master Problem (RMP), denoted $MP(\Omega^1)$:

$$\left\{ \begin{array}{l} \max \sum_{p=1}^{N_V} \sum_{r \in \Omega_p^1} g(r) \cdot \theta_r \\ \text{s.t. } \sum_{p=1}^{N_V} \sum_{r \in \Omega_p^1} a_{ri} \theta_r \leq 1, \forall i = 1, \dots, N_W, \\ \quad \sum_{r \in \Omega_p^1} \theta_r \leq 1, \forall p = 1, \dots, N_V, \\ \quad \sum_{p=1}^{N_V} \sum_{r \in \Omega_p^1(j_k)} t_r(j_k) \theta_r - \sum_{p=1}^{N_V} \sum_{r \in \Omega_p^1(i_k)} t_r(i_k) \theta_r \leq -t_{D_k}, \\ \hspace{10cm} \forall k = 1, \dots, N_D, \\ \quad \sum_{p=1}^{N_V} \sum_{r \in \Omega_p^1(i_k)} \theta_r = 1, \forall k = 1, \dots, N_D, \\ \quad \sum_{p=1}^{N_V} \sum_{r \in \Omega_p^1(j_k)} \theta_r = 1, \forall k = 1, \dots, N_D, \\ \quad \theta_r \geq 0, \forall r \in \Omega^1. \end{array} \right. \quad (13)$$

Column generation (8/11)

- The dual program of (13), denoted by $D(\Omega^1)$:

$$\left\{ \begin{array}{l} \min \sum_{i=1}^{N_W} \lambda_i + \sum_{p=1}^{N_V} \mu_p - \sum_{k=1}^{N_D} t_{D_k} \cdot \alpha_k \\ \text{s.t.} \sum_{i=1}^{N_W} a_{ri} \lambda_i + \mu_p + \sum_{k=1}^{N_D} (t_r(j_k) \cdot a_{r,j_k} - t_r(i_k) \cdot a_{r,i_k}) \alpha_k \geq g(r), \\ \lambda_i \geq 0, \forall i \in N_W \setminus \mathbf{D}, \\ \lambda_i \in \mathbb{R}, \forall i \in \mathbf{D}, \\ \mu_p \geq 0, \forall p = 1, \dots, N_V, \\ \alpha_k \geq 0, \forall k = 1, \dots, N_D. \end{array} \right. \quad \forall r \in \Omega_p^1, p \in N_V, \quad (14)$$

λ_i : is the dual variable related to the visitation constraint of waypoint i

μ_p : is the dual variable related to the implementation constraint of vehicle p.

α_k : is the dual variable corresponding to the dependency constraint.

Column generation (9/11)

- Suppose that $(\bar{\lambda}, \bar{\mu}, \bar{\alpha}) = (\bar{\lambda}_1, \dots, \bar{\lambda}_{N_W}, \bar{\mu}_1, \dots, \bar{\mu}_{N_V}, \bar{\alpha}_1, \dots, \bar{\alpha}_{N_D})$ is an optimal solution of the dual problem $D(\Omega^1)$, we have

$$\sum_{i=1}^{N_W} a_{ri} \bar{\lambda}_i + \bar{\mu}_p + \sum_{k=1}^{N_D} (t_r(j_k) \cdot a_{r,j_k} - t_r(i_k) \cdot a_{r,i_k}) \bar{\alpha}_k \geq g(r), \forall r \in \Omega_p^1, p \in N_V. \quad (15)$$

If this condition (15) holds for all $r \in \Omega_p, p \in N_V$

➡ $(\bar{\lambda}, \bar{\mu}, \bar{\alpha})$ is also the optimal solution of the dual program of (MP).

If not ➡ we will look for a trajectory $r \in \Omega_p \setminus \Omega_p^1$ for an UAV $p \in N_V$ such that :

$$\sum_{i=1}^{N_W} a_{ri} \bar{\lambda}_i + \bar{\mu}_p + \sum_{k=1}^{N_D} (t_r(j_k) \cdot a_{r,j_k} - t_r(i_k) \cdot a_{r,i_k}) \bar{\alpha}_k < g(r) \quad (16)$$

➡ The sub-problem

Column generation (10/11)

The sub-problem :

- Solving the sub-problem leads to solving a problem which is quite similar to (11), but with only one vehicle p so as to find a trajectory r which satisfies the condition (16):

Subject to :

[dynamic system]

[avoidance the obstacles]

[dependency constraints] : modified

$$\sum_{t=1}^{N_T} tb_{i_k,t} + R(1 - \sum_{t=1}^{N_T} b_{i_k,t}) \geq t_{D_k} + \sum_{t=1}^{N_T} tb_{j_k,t}.$$

⇒ The sub-problem is easier to solve than the original problem (11):

- The set of compatible waypoints : **sub-set**.

Column generation (11/11)

Column generation-based algorithm :

Step 1. Generate initial sets Ω_p^1 for each vehicle $p \in N_v$,

Step 2. Solve the problem (13) in order to obtain the optimal solution and its dual solution $(\bar{\lambda}, \bar{\mu}, \bar{\alpha})$,

Step 3. For each vehicle $p \in N_v$, solving the sub-problem optimally to find a trajectory $r \in \Omega_k \setminus \Omega_k^1$ satisfying the condition (16) and update

$$\Omega_k^1 := \Omega_k^1 \cup \{r\},$$

Step 4. Iterate step 2-3 until there is no trajectory satisfying the condition.

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Branching (1/4)

If θ_r is 0-1 \rightarrow the solution θ is optimal.

If θ_r is fractional \rightarrow Branching : 2 techniques
- Branching 1: there exists a pair (i, p) , i is a waypoint and p is an UAV such that:

$$0 < \sum_{r \in \Omega_p^1} a_{ri} \theta_r < 1$$

Left

Branching

Right

$$\sum_{r \in \Omega_p^1} a_{ri} \theta_r = 1$$

$$\sum_{r \in \Omega_p^1} a_{ri} \theta_r = 0$$

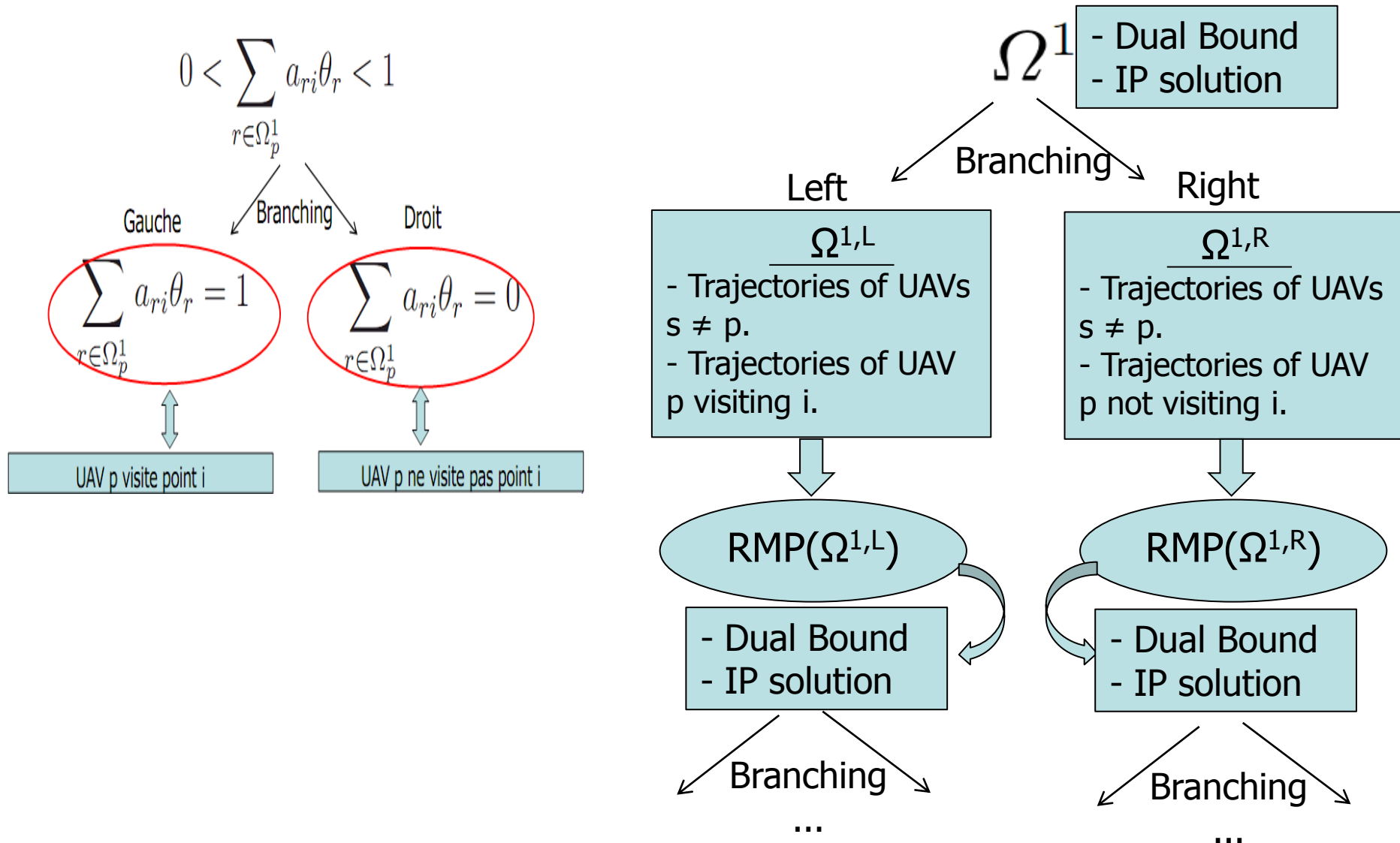


UAV p must visit waypoint i



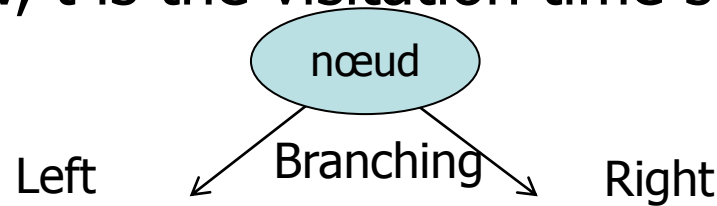
UAV p does not visit waypoint i

Branching (2/4)



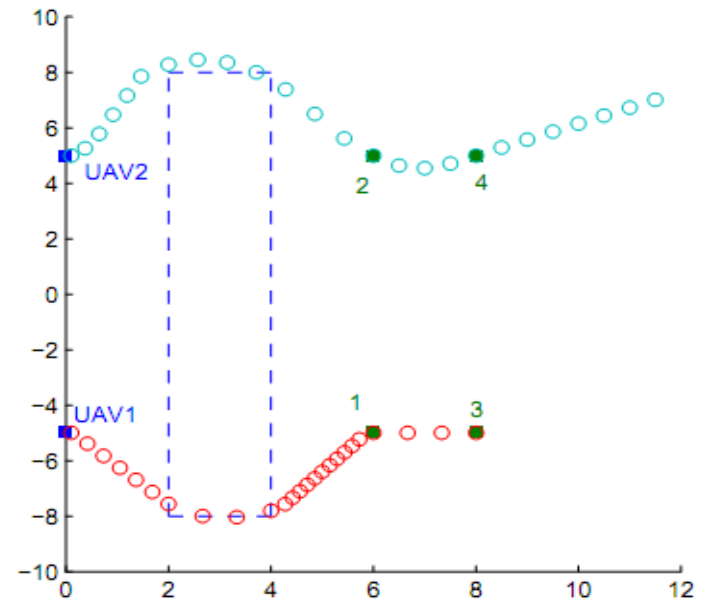
Branching (3/4)

- Branching 2: there exist a triple (i, p, t) , i is a waypoint and p is an UAV, t is the visitation time such that:



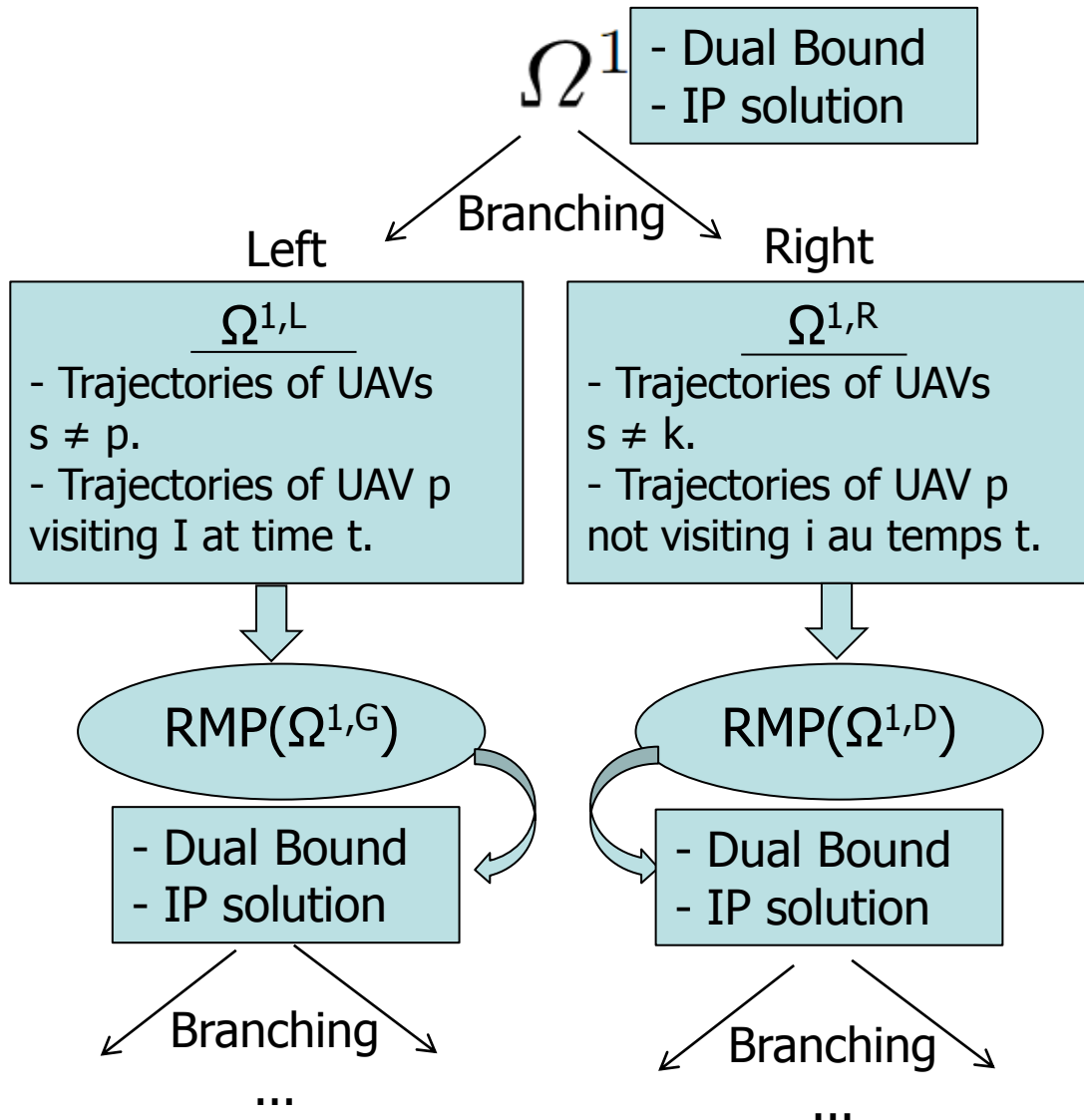
UAV p visits waypoint i exactly at time t

UAV p does not visit waypoint i exactly at time t

$$\begin{pmatrix} \begin{array}{cccccccccccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ \hline -22 & 14 & 20 & 0 & -10 & -23 & 19 & 13 & 18 & 0 & -10 & -23 \\ -25 & 18 & 22 & -10 & 17 & 13 & -12 & -23 & -25 & -25 & 0 & 0 \\ \hline 22 & 0 & 0 & 0 & 10 & 23 & 0 & 0 & 0 & 0 & 10 & 23 \\ 0 & 14 & 20 & 0 & 0 & 0 & 19 & 13 & 18 & 0 & 0 & 0 \\ 25 & 0 & 0 & 23 & 0 & 0 & 12 & 23 & 25 & 25 & 0 & 0 \\ 0 & 18 & 22 & 13 & 17 & 13 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \end{pmatrix}$$


$\theta = (0; 0; 0; 0; 0; 0; 1; 0.5384615; 0; 0.4615385; 0; 0; 0)$

Branching (4/4)



- Update the best IP solution (Best_Obj).
- Eliminate the nodes providing dual bound $<$ Best_Obj

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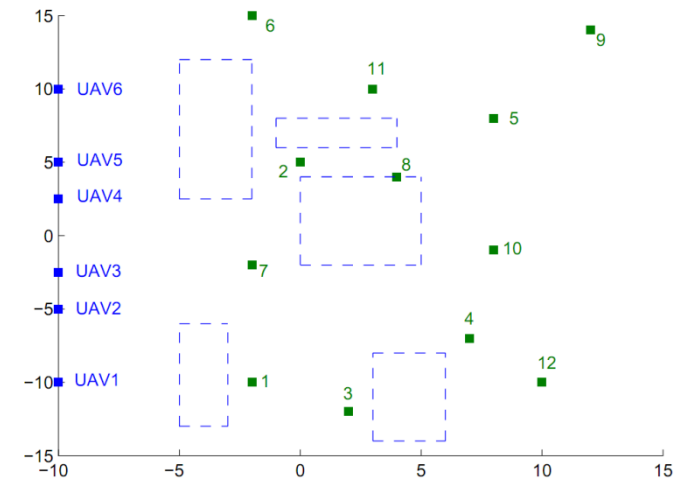
Simulation: Data 1

UAV	mass (kg)	Initial velocity		ω_{max} ($^{\circ}/s$)	v_{max} (m/s)	f_{max} (N)	F (N)
		X(m/s)	Y(m/s)				
1	5	0.1	0	15	1.5	1.9635	25
2	5	0.1	0	15	1.5	1.9635	25
3	5	0.1	0	15	1.0	1.3090	25
4	5	0.1	0	15	1.5	1.9635	25
5	5	0.1	0	15	1.0	1.3090	25
6	5	0.1	0	15	1.0	1.3090	25

5760 binary variables
1446 continuous variables
16116 constraints

$$t_6 - t_9 \geq 1$$

$$t_1 - t_7 \geq 2$$



Prob

CPLEX

Column generation

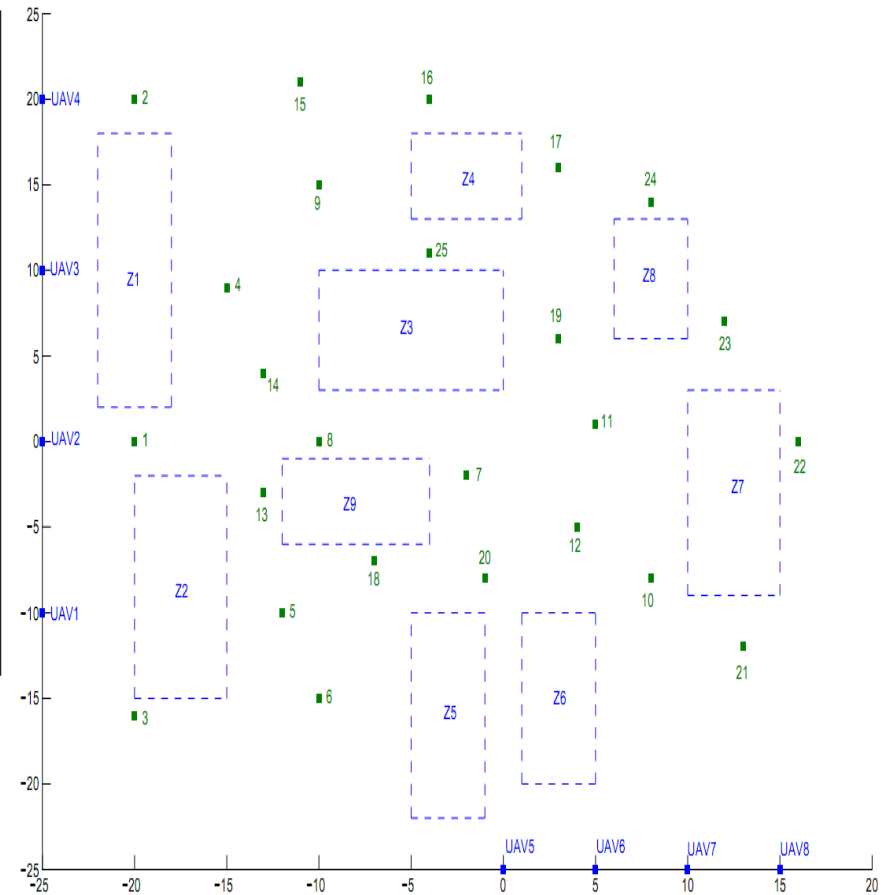
	LP relaxation	Objective	Time(s)	Dual Bound	IP solution	Optimal	Time(s)	Nodes	Depth
1	211.992952	203.858889	3600	205.857072	205.855858	205.855858	185.11	3	1
2	196.991954	178.852894	3600	189.839830	189.839826	189.839826	134.10	1	0
3	200.990944	194.858894	98.09	194.860893	194.856896	194.858894	293.14	3	1
4	218.990941	194.866876	3600	210.860849	210.860849	210.860849	123.16	1	0
5	202.992953	179.865901	3600	190.873856	190.863855	190.873856	348.59	11	5
6	194.991954	185.861882	3600	190.852861	190.848847	190.849849	532.42	11	4
7	197.990951	185.852879	3600	192.852859	192.850859	192.850859	262.08	5	2
8	215.993944	199.869905	3600	203.372870	201.869867	202.873872	866.48	17	5
9	215.992949	195.861886	3600	201.850853	201.840855	201.850853	498.75	13	6
10	219.992940	208.846897	3600	217.372772	214.856853	214.856853	271.92	5	2

Simulation: Data 2

UAV	mass (kg)	Initial velocity		ω_{max} ($^{\circ}/s$)	v_{max} (m/s)	f_{max} (N)	F (N)
		X(m/s)	Y(m/s)				
1	5	0.1	0	15	1.5	1.9635	30
2	5	0.1	0	15	1.5	1.9635	30
3	5	0.1	0	15	2.0	2.6180	30
4	5	0.1	0	15	1.5	1.9635	30
5	5	0	0.1	15	2.0	2.6180	30
6	5	0	0.1	15	2.0	2.6180	30
7	5	0	0.1	15	2.0	2.6180	30
8	5	0.1	0.1	15	2.0	1.9635	30

12200 binary variables
 1608 continuous variables
 32448 constraints

i_k	j_k	t_{D_k}
9	16	1
13	5	2
7	8	2
25	19	4
11	22	6



Simulation: Data 2 (cont.)

Prob	CPLEX			Column generation					
	LP relaxation	Objective	Time(s)	Dual Bound	IP solution	Optimal	Time(s)	Nodes	Depth
1	403.977926	214.821828	10800	283.860815	259.814764	262.823769	5770.24	197	11
2	412.980914	258.827792	10800	272.720970	258.823776	258.829777	3910.47	111	10
3	419.981909	293.833806	10800	324.427904	304.812780	314.817780	3635.91	89	12
4	409.977917	290.822784	10800	314.258042	297.814766	308.817762	1155.81	21	6
5	414.981904	284.808804	10800	288.494062	278.814781	285.810781	364.64	7	2
6	410.982910	275.817810	10800	290.433635	272.816770	277.815770	5267.93	101	9
7	439.981918	267.821817	10800	315.997591	285.816777	293.817771	4651.38	95	12
8	433.979908	271.825806	10800	315.708516	314.809763	314.810760	348.93	5	2
9	438.980907	296.821792	10800	320.794631	311.808781	314.812761	799.59	15	4
10	434.982908	279.825797	10800	304.881974	287.811773	294.809771	1179.29	37	9

2708.42s

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Conclusion and perspectives

- Present the problem of coordination and control multiple UAVs.
- Propose a branch-and-price algorithm to solve the addressed problem.
- Compare numerical results obtained by our approach with those given by CPLEX.

Perspectives :

- Study and analyze other methods to build UAV trajectory
- Parallelize the trajectory search for all drones to speed up the computation time.

THANK YOU FOR YOUR ATTENTION!